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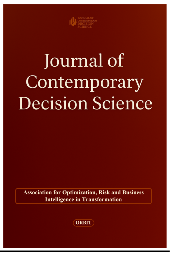

A Novel Interval-Valued Intuitionistic Fuzzy Distance Measure Incorporating Min/Max Interaction Terms

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ABSTRACT

Interval-valued intuitionistic fuzzy sets (IVIFSs) are widely employed in decision-making and pattern recognition, where distance measures serve as fundamental tools. However, many existing distance measures for IVIFSs have two limitations: one is the low discriminant ability, which usually produces the same value for different pairs; the second is the violation of the axiom of regularity, and the distance value is not within [0,1]. To overcome these shortcomings, this paper proposes a new distance measure, which integrates a non-linear transformation of endpoint differences with min/max interaction terms. In addition, it is proved that the proposed measure satisfies all four axioms of distance. Finally, comparative experiments show the effectiveness and superiority of the proposed measure. The results establish the proposed measure as a more discriminative and axiomatically sound tool for IVIFS-based analysis.

Keywords:
Interval-valued intuitionistic fuzzy set; Distance measure; Decision-making

1. Introduction

Interval-valued intuitionistic fuzzy sets (IVIFSs) proposed by Atanasov and Gargov [1] is an extension of intuitionistic fuzzy sets (IFSs) [2], which provides a strong mathematical framework for capturing uncertainty and fuzziness in complex decision-making environments. By representing membership and non-membership as intervals rather than a single value, IVIFSs can more faithfully reflect the inherent hesitation and fluctuation in human judgment, making it particularly suitable for dealing with real-world problems characterized by inaccurate and fuzzy information. Therefore, IVIFSs have been widely used in multi-attribute decision-making [3, 4], pattern recognition [5, 6], risk assessment [7, 8] and other fields.

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In IVIFSs-based analysis tools, distance measures play a crucial role in quantifying the dissimilarity between two IVIFSs, enabling comparative evaluation, clustering, and ranking [9]. Xu and Chen [10] provided a basic overview of distance and similarity measures for IFSs and IVIFSs, systematically classified existing methods, and lay the foundation for subsequent development in this field. Over the past decade, numerous distance measures have been proposed and refined. For example, Tiwari and Gupta [11] studied entropy, distance and similarity measures in interval-valued intuitionistic fuzzy environments, proposed new formulas, and enhanced the theoretical framework. Rashid et al. [12] proposed a distance-based entropy measure for IVIFSs and proved its practicability in multi-attribute decision-making problems. Li et al. [13] studied the thickness relationship and structural difference in interval-valued intuitionistic fuzzy granular space by proposing a pair of granular structure distances. More recently, Gohain et al. [5] introduced a distance measure from an optimistic viewpoint, capturing the positive aspects of information in IVIFSs, and validated its effectiveness in pattern recognition and clustering problems. Rani et al. [14] introduced a distance measure for IVIFSs and integrated it with the “RANCOM-WISP” methodology for offshore wind power station location selection, showcasing the applicability of such measures in renewable energy planning. Mishra et al. [15] further advanced the field by incorporating interval-valued intuitionistic fuzzy information into a combined compromise solution approach to assess collaborative innovation system challenges in public higher education. Mishra et al. [16] applied the interval-valued intuitionistic fuzzy gained and lost dominance score methodology to household solid waste processing plant location selection, demonstrating the versatility of distance-based approaches in environmental management decisions. Despite these advances, several limitations persist in existing distance measures for IVIFSs. A significant number of them exhibit insufficient discriminative ability, often producing identical distance values for distinct IVIFS pairs. Moreover, some measures violate the fundamental regularity axiom (D1) (see Section 4), yielding values outside the unit interval [0,1]—a clear breach of the axiomatic requirements for a distance measure. These shortcomings undermine their reliability in critical applications such as pattern recognition and decision-making, where precise differentiation and metric validity are essential.

To address these gaps, this paper proposes a novel distance measure for IVIFSs. The proposed measure integrates endpoint differences of membership and non-membership intervals with two interaction terms based on min/max operators, thereby capturing both the magnitude of deviations and the relative ordering of the two sets. We rigorously prove that the new measure satisfies all four axioms (D1)–(D4) and provide its reduced form for ordinary IFSs. The practical utility of the proposed measure is demonstrated through a consumer preference analysis for beverage selection, a domain where uncertainty and hesitancy are particularly pronounced. By applying the measure to identify the most preferred beverage among six alternatives, we illustrate its superior discriminative ability compared to 18 existing measures [5, 10–16].

The arrangement of this paper is as follows. Section 2 reviews the basic notions of IVIFSs and 18 representative distance measures. Section 3 introduces a new distance measure and gives a detailed proof of its axiomatic properties. Section 4 gives two comparative experiments: the first experiment examines five benchmark IVIFS pairs, and the second experiment applies the measures to a practical beverage preference decision-making problem. Section 5 summarizes the research results of this paper.

2. Preliminaries

This section reviews the basic concepts of IVIFSs, including their definitions, operational rules, and axiomatic frameworks for distance measures. In addition, 18 existing representative distance measures are listed as the benchmark for comparative analysis in subsequent sections.

Definition 1 ([1]). Let Z be a non-empty domain, and let $I([0,1])$ be the set of all closed subintervals on the interval [0,1]. Then, an IVIFS $\mathcal{G}$ on Z is defined as

$$\mathcal{G} = \{\langle z, \widetilde{\mu}_{\mathcal{G}}(z), \widetilde{\nu}_{\mathcal{G}}(z)\rangle | z \in Z\},$$

where the functions $\widetilde{\mu}_{\mathcal{G}} : Z \to I([0,1])$ and $\widetilde{\nu}_{\mathcal{G}} : Z \to I([0,1])$ denote the membership degree and non-membership degree of the element z in $\mathcal{G}$, respectively, with the condition $(\forall z \in Z) \max \widetilde{\mu}_{\mathcal{G}}(z) + \max \widetilde{\nu}_{\mathcal{G}}(z) \leq 1$. In general, $(\forall z \in Z) \min \widetilde{\mu}_{\mathcal{G}}(z)$, $\max \widetilde{\mu}_{\mathcal{G}}(z)$, $\min \widetilde{\nu}_{\mathcal{G}}$ and $\max \widetilde{\nu}_{\mathcal{G}}(z)$ are denoted as $\mu_{\mathcal{G}}^{-}(z)$, $\mu_{\mathcal{G}}^{+}(z)$, $\nu_{\mathcal{G}}^{-}(z)$ and $\nu_{\mathcal{G}}^{+}(z)$, respectively. For each $z \in Z$, the symbol $\widetilde{\pi}_{\mathcal{G}}(z) = [\pi_{\mathcal{G}}^{-}(z), \pi_{\mathcal{G}}^{+}(z)]$ is called the hesitation degree of z in $\mathcal{G}$, where $\pi_{\mathcal{G}}^{-}(z) = 1 - \mu_{\mathcal{G}}^{+}(z) - \nu_{\mathcal{G}}^{+}(z)$ and $\pi_{\mathcal{G}}^{+}(z) = 1 - \mu_{\mathcal{G}}^{-}(z) - \nu_{\mathcal{G}}^{-}(z)$.

For convenience, the set of all IVIFSs on Z is denoted by $\mathbf{IVIFS}(Z)$, where Z is a non-empty domain.

Definition 2 ([1]). Let $\mathcal{G}, \mathcal{M} \in \mathbf{IVIFS}(Z)$, then the following operations are defined:

(1) $\mathcal{G} \subseteq \mathcal{M} \Longleftrightarrow (\forall z \in Z)\ \mu_{\mathcal{G}}^{-}(z) \leq \mu_{\mathcal{M}}^{-}(z),\ \mu_{\mathcal{G}}^{+}(z) \leq \mu_{\mathcal{M}}^{+}(z),\ \nu_{\mathcal{G}}^{-}(z) \geq \nu_{\mathcal{M}}^{-}(z)$ and $\nu_{\mathcal{G}}^{+}(z) \geq \nu_{\mathcal{M}}^{+}(z)$.

(2) $\mathcal{G} = \mathcal{M} \Longleftrightarrow \mathcal{G} \subseteq \mathcal{M}$ and $\mathcal{M} \subseteq \mathcal{G}$.

(3) $\mathcal{G}^{C} = \{\langle z, [\nu_{\mathcal{G}}^{-}(z), \nu_{\mathcal{G}}^{+}(z)], [\mu_{\mathcal{G}}^{-}(z), \mu_{\mathcal{G}}^{+}(z)]\rangle | z \in Z\}$.

Definition 3 ([10]). A function $\mathscr{D}$: $\mathbf{IVIFS}(Z) \times \mathbf{IVIFS}(Z) \to [0,1]$ is a distance measure for IVIFSs if $\mathscr{D}$ meets the following conditions: $(\forall \mathcal{G}, \mathcal{M} \in \mathbf{IVIFS}(Z))$

(D1) **Regularity:** $0 \leq \mathscr{D}(\mathcal{G}, \mathcal{M}) \leq 1$.

(D2) **Reflexivity:** $\mathscr{D}(\mathcal{G}, \mathcal{M}) = 0 \Longleftrightarrow \mathcal{G} = \mathcal{M}$.

(D3) **Symmetry:** $\mathscr{D}(\mathcal{G}, \mathcal{M}) = \mathscr{D}(\mathcal{M}, \mathcal{G})$.

(D4) **Transitivity:** If there exists $\mathcal{N} \in \mathbf{IVIFS}(Z)$ such that $\mathcal{G} \subseteq \mathcal{M} \subseteq \mathcal{N}$, then $\mathscr{D}(\mathcal{G}, \mathcal{M}) \leq \mathscr{D}(\mathcal{G}, \mathcal{N})$ and $\mathscr{D}(\mathcal{M}, \mathcal{N}) \leq \mathscr{D}(\mathcal{G}, \mathcal{N})$.

- The normalized Hamming distance measure [10]:

$$\mathscr{D}_{NH}(\mathcal{G}, \mathcal{M}) = \frac{1}{4|Z|} \sum_{z \in Z} \begin{Bmatrix} |\mu_{\mathcal{G}}^{-}(z) - \mu_{\mathcal{M}}^{-}(z)| + |\mu_{\mathcal{G}}^{+}(z) - \mu_{\mathcal{M}}^{+}(z)| + \\ |\nu_{\mathcal{G}}^{-}(z) - \nu_{\mathcal{M}}^{-}(z)| + |\nu_{\mathcal{G}}^{+}(z) - \nu_{\mathcal{M}}^{+}(z)| \end{Bmatrix}. \tag{1}$$

- The normalized Euclidean distance measure [10]:

$$\mathscr{D}_{NE}(\mathcal{G}, \mathcal{M}) = \left\{ \frac{1}{4|Z|} \sum_{z \in Z} \begin{bmatrix} (\mu_{\mathcal{G}}^{-}(z) - \mu_{\mathcal{M}}^{-}(z))^2 + (\mu_{\mathcal{G}}^{+}(z) - \mu_{\mathcal{M}}^{+}(z))^2 + \\ (\nu_{\mathcal{G}}^{-}(z) - \nu_{\mathcal{M}}^{-}(z))^2 + (\nu_{\mathcal{G}}^{+}(z) - \nu_{\mathcal{M}}^{+}(z))^2 \end{bmatrix} \right\}^{1/2}. \tag{2}$$

- The normalized Hamming distance measure based on Hausdorff metric [10]:

$$\mathscr{D}_{NHH}(\mathcal{G}, \mathcal{M}) = \frac{1}{|Z|} \sum_{z \in Z} \max \begin{Bmatrix} |\mu_{\mathcal{G}}^{-}(z) - \mu_{\mathcal{M}}^{-}(z)|, |\mu_{\mathcal{G}}^{+}(z) - \mu_{\mathcal{M}}^{+}(z)|, \\ |\nu_{\mathcal{G}}^{-}(z) - \nu_{\mathcal{M}}^{-}(z)|, |\nu_{\mathcal{G}}^{+}(z) - \nu_{\mathcal{M}}^{+}(z)| \end{Bmatrix}. \tag{3}$$

- The normalized Euclidean distance measure based on Hausdorff metric [10]:

$$\mathscr{D}_{NEH}(\mathcal{G},\mathcal{M})=\left\{\frac{1}{|Z|}\sum_{z\in Z}\max\left\{\begin{array}{l}(\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z))^{2},(\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z))^{2},\\(\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z))^{2},(\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z))^{2}\end{array}\right\}\right\}^{1/2}. \tag{4}$$

- Tiwari and Gupta [11]'s distance measures: For $p\geq 2$,

$$\mathscr{D}_{TG1}(\mathcal{G},\mathcal{M})=\left\{\frac{1}{12|Z|}\sum_{z\in Z}\left[\begin{array}{l}(\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z))^{2}+(\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z))^{2}+\\(\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z))^{2}+(\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z))^{2}+\\(\pi_{\mathcal{G}}^{-}(z)-\pi_{\mathcal{M}}^{-}(z))^{2}+(\pi_{\mathcal{G}}^{+}(z)-\pi_{\mathcal{M}}^{+}(z))^{2}\end{array}\right]\right\}^{1/2}, \tag{5}$$

$$\mathscr{D}_{TG2}(\mathcal{G},\mathcal{M})=\frac{1}{8|Z|}\sum_{z\in Z}\left\{\begin{array}{l}|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|+|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|+\\|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|+|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|+\\|\pi_{\mathcal{G}}^{-}(z)-\pi_{\mathcal{M}}^{-}(z)|+|\pi_{\mathcal{G}}^{+}(z)-\pi_{\mathcal{M}}^{+}(z)|\end{array}\right\}, \tag{6}$$

$$\mathscr{D}_{TG3}(\mathcal{G},\mathcal{M})=\frac{1}{4|Z|}\sum_{z\in Z}\left\{\begin{array}{l}\max\{|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|,|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|\}+\\\max\{|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|,|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|\}+\\\max\{|\pi_{\mathcal{G}}^{-}(z)-\pi_{\mathcal{M}}^{-}(z)|+|\pi_{\mathcal{G}}^{+}(z)-\pi_{\mathcal{M}}^{+}(z)|\}\end{array}\right\}, \tag{7}$$

$$\mathscr{D}_{TG4}(\mathcal{G},\mathcal{M})=\frac{1}{4|Z|}\sum_{z\in Z}\max\left\{\begin{array}{l}\frac{1}{2}\left(|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|+|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|\right),\\\frac{1}{2}\left(|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|+|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|\right),\\\frac{1}{2}\left(|\pi_{\mathcal{G}}^{-}(z)-\pi_{\mathcal{M}}^{-}(z)|+|\pi_{\mathcal{G}}^{+}(z)-\pi_{\mathcal{M}}^{+}(z)|\right)\end{array}\right\}, \tag{8}$$

$$\mathscr{D}_{TG5}(\mathcal{G},\mathcal{M})=\frac{1}{2|Z|}\sum_{z\in Z}\left\{\begin{array}{l}\frac{1}{8}\left\{\begin{array}{l}|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|+|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|+\\|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|+|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|+\\|\pi_{\mathcal{G}}^{-}(z)-\pi_{\mathcal{M}}^{-}(z)|+|\pi_{\mathcal{G}}^{+}(z)-\pi_{\mathcal{M}}^{+}(z)|\end{array}\right\}+\\\frac{1}{4}\max\left\{\begin{array}{l}\left(|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|+|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|\right),\\\left(|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|+|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|\right),\\\left(|\pi_{\mathcal{G}}^{-}(z)-\pi_{\mathcal{M}}^{-}(z)|+|\pi_{\mathcal{G}}^{+}(z)-\pi_{\mathcal{M}}^{+}(z)|\right)\end{array}\right\}\end{array}\right\}, \tag{9}$$

$$\mathscr{D}_{TG6}(\mathcal{G},\mathcal{M})=\left\{\frac{1}{12|Z|}\sum_{z\in Z}\left\{\begin{array}{l}(\max\{|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|,|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|\})^{p}+\\(\max\{|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|,|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|\})^{p}+\\(\max\{|\pi_{\mathcal{G}}^{-}(z)-\pi_{\mathcal{M}}^{-}(z)|,|\pi_{\mathcal{G}}^{+}(z)-\pi_{\mathcal{M}}^{+}(z)|\})^{p}\end{array}\right\}\right\}^{1/p}. \tag{10}$$

- Rashid et al. [12]'s distance measure:

$$\mathscr{D}_{R}(\mathcal{G},\mathcal{M})=\frac{1}{2|Z|}\sum_{z\in Z}\left\{\begin{array}{l}\min\{|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|,|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|,|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|,|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|\quad\}+\\\max\{|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|,|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|,|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|,|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|\quad\}\end{array}\right\}. \tag{11}$$

- Li et al. [13]'s distance measures:

$$\mathscr{D}_{L1}(\mathcal{G},\mathcal{M})=\frac{1}{2|Z|}\sum_{z\in Z}\left\{|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|+\max\{|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|,|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|,|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|\}\right\}. \tag{12}$$

$$\mathscr{D}_{L2}(\mathcal{G},\mathcal{M})=\frac{1}{2|Z|}\sum_{z\in Z}\left\{|\nu_{\mathcal{G}}^{-}(z)-\nu_{\mathcal{M}}^{-}(z)|+\max\{|\mu_{\mathcal{G}}^{-}(z)-\mu_{\mathcal{M}}^{-}(z)|,|\mu_{\mathcal{G}}^{+}(z)-\mu_{\mathcal{M}}^{+}(z)|,|\nu_{\mathcal{G}}^{+}(z)-\nu_{\mathcal{M}}^{+}(z)|\}\right\}. \tag{13}$$

- Gohain et al. [5]'s distance measure:

$$\mathscr{D}_G(\mathcal{G},\mathcal{M}) = \frac{1}{6|Z|}\sum_{z\in Z}\left\{\begin{array}{l} \dfrac{|\mu_{\mathcal{G}}^-(z)-\mu_{\mathcal{M}}^-(z)|+|\nu_{\mathcal{G}}^-(z)-\nu_{\mathcal{M}}^-(z)|}{1+\mu_{\mathcal{G}}^-(z)+\mu_{\mathcal{M}}^-(z)+(1-\nu_{\mathcal{G}}^-(z))(1-\nu_{\mathcal{M}}^-(z))}+ \\ \dfrac{|\mu_{\mathcal{G}}^+(z)-\mu_{\mathcal{M}}^+(z)|+|\nu_{\mathcal{G}}^+(z)-\nu_{\mathcal{M}}^+(z)|}{1+\mu_{\mathcal{G}}^+(z)+\mu_{\mathcal{M}}^+(z)+(1-\nu_{\mathcal{G}}^+(z))(1-\nu_{\mathcal{M}}^+(z))}+ \\ |\min\{\mu_{\mathcal{G}}^-(z),\nu_{\mathcal{M}}^-(z)\}-\min\{\mu_{\mathcal{M}}^-(z),\nu_{\mathcal{G}}^-(z)\}|+ \\ |\min\{\mu_{\mathcal{G}}^+(z),\nu_{\mathcal{M}}^+(z)\}-\min\{\mu_{\mathcal{M}}^+(z),\nu_{\mathcal{G}}^+(z)\}|+ \\ |\max\{\mu_{\mathcal{G}}^-(z),\nu_{\mathcal{M}}^-(z)\}-\max\{\mu_{\mathcal{M}}^-(z),\nu_{\mathcal{G}}^-(z)\}|+ \\ |\max\{\mu_{\mathcal{G}}^+(z),\nu_{\mathcal{M}}^+(z)\}-\max\{\mu_{\mathcal{M}}^+(z),\nu_{\mathcal{G}}^+(z)\}| \end{array}\right\}. \tag{14}$$

- Rani et al. [14]'s distance measures:

$$\mathscr{D}_{R1}(\mathcal{G},\mathcal{M}) = \frac{1}{2|Z|}\sum_{z\in Z}\left\{\begin{array}{l} \min\{1,|\mu_{\mathcal{G}}^-(z)-\mu_{\mathcal{M}}^-(z)|+|\nu_{\mathcal{G}}^-(z)-\nu_{\mathcal{M}}^-(z)|\}+ \\ \min\{1,|\mu_{\mathcal{G}}^+(z)-\mu_{\mathcal{M}}^+(z)|+|\nu_{\mathcal{G}}^+(z)-\nu_{\mathcal{M}}^+(z)|\} \end{array}\right\}. \tag{15}$$

$$\mathscr{D}_{R2}(\mathcal{G},\mathcal{M}) = \frac{1}{|Z|}\sum_{z\in Z}\left\{\begin{array}{l} |\mu_{\mathcal{G}}^-(z)-\mu_{\mathcal{M}}^-(z)|+|\nu_{\mathcal{G}}^-(z)-\nu_{\mathcal{M}}^-(z)|+|\mu_{\mathcal{G}}^+(z)-\mu_{\mathcal{M}}^+(z)|+|\nu_{\mathcal{G}}^+(z)-\nu_{\mathcal{M}}^+(z)| \\ -|\mu_{\mathcal{G}}^-(z)-\mu_{\mathcal{M}}^-(z)|\cdot|\nu_{\mathcal{G}}^-(z)-\nu_{\mathcal{M}}^-(z)|-|\mu_{\mathcal{G}}^+(z)-\mu_{\mathcal{M}}^+(z)|\cdot|\nu_{\mathcal{G}}^+(z)-\nu_{\mathcal{M}}^+(z)| \end{array}\right\}. \tag{16}$$

- Mishra et al. [15, 16]'s distance measure:

$$\mathscr{D}_{M1}(\mathcal{G},\mathcal{M}) = \frac{1}{8|Z|}\sum_{z\in Z}\left\{\begin{array}{l} |\mu_{\mathcal{G}}^-(z)-\mu_{\mathcal{M}}^-(z)|+|\mu_{\mathcal{G}}^+(z)-\mu_{\mathcal{M}}^+(z)|+|\nu_{\mathcal{G}}^-(z)-\nu_{\mathcal{M}}^-(z)|+|\nu_{\mathcal{G}}^+(z)-\nu_{\mathcal{M}}^+(z)|+ \\ |\mu_{\mathcal{G}}^-(z)\nu_{\mathcal{M}}^-(z)-\mu_{\mathcal{M}}^-(z)\nu_{\mathcal{G}}^-(z)|+|\mu_{\mathcal{G}}^+(z)\nu_{\mathcal{M}}^+(z)-\mu_{\mathcal{M}}^+(z)\nu_{\mathcal{G}}^+(z)|+ \\ |\min\{\mu_{\mathcal{G}}^-(z),\nu_{\mathcal{M}}^-(z)\}-\min\{\mu_{\mathcal{M}}^-(z),\nu_{\mathcal{G}}^-(z)\}|+ \\ |\min\{\mu_{\mathcal{G}}^+(z),\nu_{\mathcal{M}}^+(z)\}-\min\{\mu_{\mathcal{M}}^+(z),\nu_{\mathcal{G}}^+(z)\}|+ \\ |\max\{\mu_{\mathcal{G}}^-(z),\nu_{\mathcal{M}}^-(z)\}-\max\{\mu_{\mathcal{M}}^-(z),\nu_{\mathcal{G}}^-(z)\}|+ \\ |\max\{\mu_{\mathcal{G}}^+(z),\nu_{\mathcal{M}}^+(z)\}-\max\{\mu_{\mathcal{M}}^+(z),\nu_{\mathcal{G}}^+(z)\}| \end{array}\right\}. \tag{17}$$

$$\mathscr{D}_{M2}(\mathcal{G},\mathcal{M}) = \frac{1}{4|Z|}\sum_{z\in Z}\left\{\begin{array}{l} (|\mu_{\mathcal{G}}^-(z)-\mu_{\mathcal{M}}^-(z)|+|\nu_{\mathcal{G}}^-(z)-\nu_{\mathcal{M}}^-(z)|)\cdot\left(1-\dfrac{1}{2}|\pi_{\mathcal{G}}^+(z)-\pi_{\mathcal{M}}^+(z)|\right)+ \\ (|\mu_{\mathcal{G}}^+(z)-\mu_{\mathcal{M}}^+(z)|+|\nu_{\mathcal{G}}^+(z)-\nu_{\mathcal{M}}^+(z)|)\cdot\left(1-\dfrac{1}{2}|\pi_{\mathcal{G}}^-(z)-\pi_{\mathcal{M}}^-(z)|\right) \end{array}\right\}. \tag{18}$$

3. A new distance measure for IVIFSs

In this section, a new distance measure for IVIFSs is proposed. Then, we prove that the proposed measure satisfies all four axioms (D1)–(D4). Moreover, its specialization to ordinary IFSs is presented.

Definition 4. Let $\mathcal{G},\mathcal{M}\in\mathbf{IVIFS}(Z)$. Then, a function $\mathscr{D}$ is defined by

$$\mathscr{D}(\mathcal{G},\mathcal{M}) = \frac{5}{14|Z|}\sum_{z\in Z}\left\{\begin{array}{r} \dfrac{\Delta(z)}{1+\Delta(z)}+|\min\{\mu_{\mathcal{G}}^-(z),\nu_{\mathcal{M}}^+(z)\}-\min\{\mu_{\mathcal{M}}^-(z),\nu_{\mathcal{G}}^+(z)\}| \\ +|\max\{\mu_{\mathcal{G}}^+(z),\nu_{\mathcal{M}}^-(z)\}-\max\{\mu_{\mathcal{M}}^+(z),\nu_{\mathcal{G}}^-(z)\}| \end{array}\right\}, \tag{19}$$

where $\Delta(z)=|\mu_{\mathcal{G}}^-(z)-\mu_{\mathcal{M}}^-(z)|+|\mu_{\mathcal{G}}^+(z)-\mu_{\mathcal{M}}^+(z)|+|\nu_{\mathcal{G}}^-(z)-\nu_{\mathcal{M}}^-(z)|+|\nu_{\mathcal{G}}^+(z)-\nu_{\mathcal{M}}^+(z)|$.

Theorem 1. The function $\mathscr{D}$ defined by Equation (19) is a distance measure for IVIFSs.

Proof. From Definition 3, we can see that the following four cases need to be considered.

(D1) Since Definition 1, we have $(\forall z \in Z)\ \mu^-_{\mathcal{G}}(z), \mu^+_{\mathcal{G}}(z), \nu^-_{\mathcal{G}}(z), \nu^+_{\mathcal{G}}(z), \mu^-_{\mathcal{M}}(z), \mu^+_{\mathcal{M}}(z), \nu^-_{\mathcal{M}}(z), \nu^+_{\mathcal{M}}(z) \in [0,1]$ and so $(\forall z \in Z)$

$$\begin{aligned}
&0 \leq |\mu^-_{\mathcal{G}}(z) - \mu^-_{\mathcal{M}}(z)| \leq 1,\\
&0 \leq |\mu^+_{\mathcal{G}}(z) - \mu^+_{\mathcal{M}}(z)| \leq 1,\\
&0 \leq |\nu^-_{\mathcal{G}}(z) - \nu^-_{\mathcal{M}}(z)| \leq 1,\\
&0 \leq |\nu^+_{\mathcal{G}}(z) - \nu^+_{\mathcal{M}}(z)| \leq 1,\\
&0 \leq \min\{\mu^-_{\mathcal{G}}(z), \nu^+_{\mathcal{M}}(z)\} - \min\{\mu^-_{\mathcal{M}}(z), \nu^+_{\mathcal{G}}(z)\}| \leq 1,\\
&0 \leq \max\{\mu^-_{\mathcal{G}}(z), \nu^+_{\mathcal{M}}(z)\} - \max\{\mu^-_{\mathcal{M}}(z), \nu^+_{\mathcal{G}}(z)\}| \leq 1.
\end{aligned}$$

Then, we have $(\forall z \in Z)$

$$0 \leq \left\{ \begin{array}{r} \dfrac{\Delta(z)}{1+\Delta(z)} + |\min\{\mu^-_{\mathcal{G}}(z), \nu^+_{\mathcal{M}}(z)\} - \min\{\mu^-_{\mathcal{M}}(z), \nu^+_{\mathcal{G}}(z)\}| \\ + |\max\{\mu^+_{\mathcal{G}}(z), \nu^-_{\mathcal{M}}(z)\} - \max\{\mu^+_{\mathcal{M}}(z), \nu^-_{\mathcal{G}}(z)\}| \end{array} \right\} \leq \frac{14}{5}.$$

Therefore, the condition $0 \leq \mathscr{D}(\mathcal{G}, \mathcal{M}) \leq 1$ holds.

(2) Clearly,

$$\begin{aligned}
\mathscr{D}(\mathcal{G}, \mathcal{M}) = 0 &\iff \frac{5}{14|Z|} \sum_{z \in Z} \left\{ \begin{array}{r} \dfrac{\Delta(z)}{1+\Delta(z)} + |\min\{\mu^-_{\mathcal{G}}(z), \nu^+_{\mathcal{M}}(z)\} - \min\{\mu^-_{\mathcal{M}}(z), \nu^+_{\mathcal{G}}(z)\}| \\ + |\max\{\mu^+_{\mathcal{G}}(z), \nu^-_{\mathcal{M}}(z)\} - \max\{\mu^+_{\mathcal{M}}(z), \nu^-_{\mathcal{G}}(z)\}| \end{array} \right\} = 0 \\
&\iff (\forall z \in Z) \ \left\{ \begin{array}{r} \dfrac{\Delta(z)}{1+\Delta(z)} + |\min\{\mu^-_{\mathcal{G}}(z), \nu^+_{\mathcal{M}}(z)\} - \min\{\mu^-_{\mathcal{M}}(z), \nu^+_{\mathcal{G}}(z)\}| \\ + |\max\{\mu^+_{\mathcal{G}}(z), \nu^-_{\mathcal{M}}(z)\} - \max\{\mu^+_{\mathcal{M}}(z), \nu^-_{\mathcal{G}}(z)\}| \end{array} \right\} = 0 \\
&\iff (\forall z \in Z) \ \left\{ \begin{array}{l} |\mu^-_{\mathcal{G}}(z) - \mu^-_{\mathcal{M}}(z)| = 0 \\ |\mu^+_{\mathcal{G}}(z) - \mu^+_{\mathcal{M}}(z)| = 0 \\ |\nu^-_{\mathcal{G}}(z) - \nu^-_{\mathcal{M}}(z)| = 0 \\ |\nu^+_{\mathcal{G}}(z) - \nu^+_{\mathcal{M}}(z)| = 0 \\ |\min\{\mu^-_{\mathcal{G}}(z), \nu^+_{\mathcal{M}}(z)\} - \min\{\mu^-_{\mathcal{M}}(z), \nu^+_{\mathcal{G}}(z)\}| = 0 \\ |\max\{\mu^+_{\mathcal{G}}(z), \nu^-_{\mathcal{M}}(z)\} - \max\{\mu^+_{\mathcal{M}}(z), \nu^-_{\mathcal{G}}(z)\}| = 0 \end{array} \right. \\
&\iff (\forall z \in Z) \ \left\{ \begin{array}{l} \mu^-_{\mathcal{G}}(z) = \mu^-_{\mathcal{M}}(z) \\ \mu^+_{\mathcal{G}}(z) = \mu^+_{\mathcal{M}}(z) \\ \nu^-_{\mathcal{G}}(z) = \nu^-_{\mathcal{M}}(z) \\ \nu^+_{\mathcal{G}}(z) = \nu^+_{\mathcal{M}}(z) \end{array} \right. \\
&\iff \mathcal{G} = \mathcal{M}
\end{aligned}$$

(D3) Clearly,

$$\begin{aligned}
\mathscr{D}(\mathcal{G},\mathcal{M}) =&\frac{5}{14|Z|}\sum_{z\in Z}\left\{\begin{aligned}\frac{\Delta(z)}{1+\Delta(z)}&+|\min\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{M}}(z)\}-\min\{\mu^-_{\mathcal{M}}(z),\nu^+_{\mathcal{G}}(z)\}|\\ &+|\max\{\mu^+_{\mathcal{G}}(z),\nu^-_{\mathcal{M}}(z)\}-\max\{\mu^+_{\mathcal{M}}(z),\nu^-_{\mathcal{G}}(z)\}|\end{aligned}\right\},\\
=&\frac{5}{14|Z|}\sum_{z\in Z}\left\{\begin{aligned}&\frac{|\mu^-_{\mathcal{G}}(z)-\mu^-_{\mathcal{M}}(z)|+|\mu^+_{\mathcal{G}}(z)-\mu^+_{\mathcal{M}}(z)|+|\nu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{M}}(z)|+|\nu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{M}}(z)|}{1+|\mu^-_{\mathcal{G}}(z)-\mu^-_{\mathcal{M}}(z)|+|\mu^+_{\mathcal{G}}(z)-\mu^+_{\mathcal{M}}(z)|+|\nu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{M}}(z)|+|\nu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{M}}(z)|}+\\ &|\min\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{M}}(z)\}-\min\{\mu^-_{\mathcal{M}}(z),\nu^+_{\mathcal{G}}(z)\}|+|\max\{\mu^+_{\mathcal{G}}(z),\nu^-_{\mathcal{M}}(z)\}-\max\{\mu^+_{\mathcal{M}}(z),\nu^-_{\mathcal{G}}(z)\}|\end{aligned}\right\}\\
=&\frac{5}{14|Z|}\sum_{z\in Z}\left\{\begin{aligned}&\frac{|\mu^-_{\mathcal{M}}(z)-\mu^-_{\mathcal{G}}(z)|+|\mu^+_{\mathcal{M}}(z)-\mu^+_{\mathcal{G}}(z)|+|\nu^-_{\mathcal{M}}(z)-\nu^-_{\mathcal{G}}(z)|+|\nu^+_{\mathcal{M}}(z)-\nu^+_{\mathcal{G}}(z)|}{1+|\mu^-_{\mathcal{M}}(z)-\mu^-_{\mathcal{G}}(z)|+|\mu^+_{\mathcal{M}}(z)-\mu^+_{\mathcal{G}}(z)|+|\nu^-_{\mathcal{M}}(z)-\nu^-_{\mathcal{G}}(z)|+|\nu^+_{\mathcal{M}}(z)-\nu^+_{\mathcal{G}}(z)|}+\\ &|\min\{\mu^-_{\mathcal{M}}(z),\nu^+_{\mathcal{G}}(z)\}-\min\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{M}}(z)\}|+|\max\{\mu^+_{\mathcal{M}}(z),\nu^-_{\mathcal{G}}(z)\}-\max\{\mu^+_{\mathcal{G}}(z),\nu^-_{\mathcal{M}}(z)\}|\end{aligned}\right\}\\
=&\mathscr{D}(\mathcal{M},\mathcal{G})
\end{aligned}$$

(D4) Let there exists $\mathcal{N}\in\mathbf{IVIFS}(Z)$ such that $\mathcal{G}\subseteq\mathcal{M}\subseteq\mathcal{N}$. Then, from Definition 2 it follows that $(\forall z\in Z)$

$$\begin{aligned}
0\le\mu^-_{\mathcal{G}}(z)\le\mu^-_{\mathcal{M}}(z)\le\mu^-_{\mathcal{N}}(z)\le 1,\\
0\le\mu^+_{\mathcal{G}}(z)\le\mu^+_{\mathcal{M}}(z)\le\mu^+_{\mathcal{N}}(z)\le 1,\\
0\le\nu^-_{\mathcal{N}}(z)\le\nu^-_{\mathcal{M}}(z)\le\nu^-_{\mathcal{G}}(z)\le 1,\\
0\le\nu^+_{\mathcal{N}}(z)\le\nu^+_{\mathcal{M}}(z)\le\nu^+_{\mathcal{G}}(z)\le 1.
\end{aligned}$$

Thus, we have $(\forall z\in Z)$

$$\begin{aligned}
|\mu^-_{\mathcal{G}}(z)-\mu^-_{\mathcal{M}}(z)|\le|\mu^-_{\mathcal{G}}(z)-\mu^-_{\mathcal{N}}(z)|,\\
|\mu^+_{\mathcal{G}}(z)-\mu^+_{\mathcal{M}}(z)|\le|\mu^+_{\mathcal{G}}(z)-\mu^+_{\mathcal{N}}(z)|,\\
|\nu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{M}}(z)|\le|\nu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{N}}(z)|,\\
|\nu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{M}}(z)|\le|\nu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{N}}(z)|.
\end{aligned}$$

Note that $(\forall z\in Z)$

$$\begin{aligned}
\min\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{N}}(z)\}\le\min\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{M}}(z)\}\le\min\{\mu^-_{\mathcal{M}}(z),\nu^+_{\mathcal{G}}(z)\}\le\min\{\mu^-_{\mathcal{N}}(z),\nu^+_{\mathcal{G}}(z)\},\\
\max\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{N}}(z)\}\le\max\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{M}}(z)\}\le\max\{\mu^-_{\mathcal{M}}(z),\nu^+_{\mathcal{G}}(z)\}\le\max\{\mu^-_{\mathcal{N}}(z),\nu^+_{\mathcal{G}}(z)\}.
\end{aligned}$$

Then, we have $(\forall z\in Z)$

$$\begin{aligned}
|\min\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{M}}(z)\}-\min\{\mu^-_{\mathcal{M}}(z),\nu^+_{\mathcal{G}}(z)\}|\le|\min\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{N}}(z)\}-\min\{\mu^-_{\mathcal{N}}(z),\nu^+_{\mathcal{G}}(z)\}|,\\
|\max\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{M}}(z)\}-\max\{\mu^-_{\mathcal{M}}(z),\nu^+_{\mathcal{G}}(z)\}|\le|\max\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{N}}(z)\}-\max\{\mu^-_{\mathcal{N}}(z),\nu^+_{\mathcal{G}}(z)\}|.
\end{aligned}$$

Therefore, we have $\mathscr{D}(\mathcal{G},\mathcal{M})\le\mathscr{D}(\mathcal{G},\mathcal{N})$. Similarly, we can deduce that $\mathscr{D}(\mathcal{M},\mathcal{N})\le\mathscr{D}(\mathcal{G},\mathcal{N})$.

□

Proposition 3.1. Let $\mathcal{G}\in\mathbf{IVIFS}(Z)$. Then, $\mathscr{D}(\mathcal{G},\mathcal{G}^C)=1$ if and only if $\mathcal{G}$ is a crisp set, that is, $(\forall z\in Z)$ either $\widetilde{\mu}_{\mathcal{G}}(z)=[1,1]$ or $\widetilde{\nu}_{\mathcal{G}}(z)=[1,1]$.

Proof. From Definitions 2 and 4 it follows that

$$\begin{aligned}
\mathscr{D}(\mathcal{G},\mathcal{G}^C)=&\frac{5}{14|Z|}\sum_{z\in Z}\left\{\begin{aligned}&\frac{|\mu^-_{\mathcal{G}}(z)-\mu^-_{\mathcal{G}^C}(z)|+|\mu^+_{\mathcal{G}}(z)-\mu^+_{\mathcal{G}^C}(z)|+|\nu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{G}^C}(z)|+|\nu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{G}^C}(z)|}{1+|\mu^-_{\mathcal{G}}(z)-\mu^-_{\mathcal{G}^C}(z)|+|\mu^+_{\mathcal{G}}(z)-\mu^+_{\mathcal{G}^C}(z)|+|\nu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{G}^C}(z)|+|\nu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{G}^C}(z)|}+\\ &|\min\{\mu^-_{\mathcal{G}}(z),\nu^+_{\mathcal{G}^C}(z)\}-\min\{\mu^-_{\mathcal{G}^C}(z),\nu^+_{\mathcal{G}}(z)\}|+|\max\{\mu^+_{\mathcal{G}}(z),\nu^-_{\mathcal{G}^C}(z)\}-\max\{\mu^+_{\mathcal{G}^C}(z),\nu^-_{\mathcal{G}}(z)\}|\end{aligned}\right\}\\
=&\frac{5}{14|Z|}\sum_{z\in Z}\left\{\frac{2|\mu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{G}}(z)|+2|\mu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{G}}(z)|}{1+2|\mu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{G}}(z)|+2|\mu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{G}}(z)|}+|\mu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{G}}(z)|+|\mu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{G}}(z)|\right\}.
\end{aligned}$$

Then, we have

$$\mathscr{D}(\mathcal{G},\mathcal{G}^C)=1 \Longleftrightarrow \frac{5}{14|Z|}\sum_{z\in Z}\left\{\frac{2|\mu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{G}}(z)|+2|\mu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{G}}(z)|}{1+2|\mu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{G}}(z)|+2|\mu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{G}}(z)|}+|\mu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{G}}(z)|+|\mu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{G}}(z)|\right\}=1$$

$$\Longleftrightarrow (\forall z\in Z)\begin{cases}|\mu^-_{\mathcal{G}}(z)-\nu^-_{\mathcal{G}}(z)|=1\\|\mu^+_{\mathcal{G}}(z)-\nu^+_{\mathcal{G}}(z)|=1\end{cases}$$

Therefore, the conclusion is established. ☐

Remark 3.1. When $\mathcal{G}$ and $\mathcal{M}$ are two IFSs on Z, Equation (19) is reduced to a distance measure of IFSs, which is shown as follows:

$$\mathscr{D}^*(\mathcal{G},\mathcal{M})=\frac{5}{14|Z|}\sum_{z\in Z}\begin{Bmatrix}\dfrac{2|\mu_{\mathcal{G}}(z)-\mu_{\mathcal{M}}(z)|+2|\nu_{\mathcal{G}}(z)-\nu_{\mathcal{M}}(z)|}{1+2|\mu_{\mathcal{G}}(z)-\mu_{\mathcal{M}}(z)|+2|\nu_{\mathcal{G}}(z)-\nu_{\mathcal{M}}(z)|}+\\|\min\{\mu_{\mathcal{G}}(z),\nu_{\mathcal{M}}(z)\}-\min\{\mu_{\mathcal{M}}(z),\nu_{\mathcal{G}}(z)\}|+\\|\max\{\mu_{\mathcal{G}}(z),\nu_{\mathcal{M}}(z)\}-\max\{\mu_{\mathcal{M}}(z),\nu_{\mathcal{G}}(z)\}|\end{Bmatrix}$$

4. Comparative discussion

This section presents a comprehensive comparison between the proposed measure and 18 existing distance measures through two complementary experiments. The first experiment, adopted from the literature [15], examines five benchmark IVIFS pairs to evaluate discriminative ability and axiomatic compliance. The second experiment applies the measures to a practical decision-making problem concerning consumer preference for beverages, demonstrating their performance in a real-world scenario.

Example 1 (Example 3.1 in [15])**.** Given five different sets of IVIFSs on $Z=\{z\}$ as follows:

S1: $\mathcal{G}_1=\{\langle z,[0.25,0.35],[0.25,0.35]\}$ and $\mathcal{M}_1=\{\langle z,[0.35,0.45],[0.35,0.45]\}$.

S2: $\mathcal{G}_2=\{\langle z,[1.00,1.00],[0.00,0.00]\}$ and $\mathcal{M}_2=\{\langle z,[0.00,0.00],[0.00,0.00]\}$.

S3: $\mathcal{G}_3=\{\langle z,[0.50,0.50],[0.50,0.50]\}$ and $\mathcal{M}_3=\{\langle z,[0.00,0.00],[0.00,0.00]\}$.

S4: $\mathcal{G}_4=\{\langle z,[0.10,0.10],[0.20,0.20]\}$ and $\mathcal{M}_4=\{\langle z,[0.30,0.30],[0.40,0.40]\}$.

S5: $\mathcal{G}_5=\{\langle z,[1.00,1.00],[0.00,0.00]\}$ and $\mathcal{M}_5=\{\langle z,[0.00,0.00],[1.00,1.00]\}$.

Table 1 summarizes the numerical results obtained by the proposed measure $\mathscr{D}$ and 18 existing distance measures on five representative pairs of IVIFSs. The comparative analysis reveals the following objective observations.

(1) Bold entries in Table 1 indicate cases where a given measure produces identical distance values for distinct IVIFS pairs. For instance:

   - The measures $\mathscr{D}_{NHH}$, $\mathscr{D}_{NEH}$, $\mathscr{D}_{TG1}-\mathscr{D}_{TG5}$, $\mathscr{D}_{L1}$, $\mathscr{D}_{R1}$ and $\mathscr{D}_{R2}$ yield identical values for the pairs $(\mathcal{G}_2,\mathcal{M}_2)$ and $(\mathcal{G}_5,\mathcal{M}_5)$.
   - The measures $\mathscr{D}_{NH}$, $\mathscr{D}_{TG2}-\mathscr{D}_{TG5}$, $\mathscr{D}_R$, $\mathscr{D}_{L2}$, $\mathscr{D}_{R1}$ and $\mathscr{D}_{M2}$ yield identical values for the pairs $(\mathcal{G}_2,\mathcal{M}_2)$ and $(\mathcal{G}_3,\mathcal{M}_3)$.

These repetitions demonstrate that many existing measures fail to discriminate between intrinsically different IVIFS pairs. In contrast, the proposed measure $\mathscr{D}$ produces distinct values across all five cases (0.1735, 0.5952, 0.2381, 0.2302, 1.0000), indicating superior discriminative capacity. Notably, only four measures ($\mathscr{D}_{NE}$, $\mathscr{D}_{TG6}$, $\mathscr{D}_{G}$, $\mathscr{D}_{M1}$, and the proposed $\mathscr{D}$) achieve this property, as indicated by the symbol "✓" in the "No ties" column.

(2) A fundamental requirement (D1) for a distance measure (see Definition 3) is that its values lie within $[0, 1]$. Several existing measures violate this axiom: $\mathscr{D}_{R2}$ exceeds 1 in multiple instances (e.g., 2.0000, 1.5000), and $\mathscr{D}_{M1}$ gives 1.2500 for the pair $(\mathcal{G}_5, \mathcal{M}_5)$. However, the proposed measure $\mathscr{D}$ strictly satisfies the axiom (D1) for every pair considered.

(3) The values of $\mathscr{D}_{TG1} - \mathscr{D}_{TG6}$ are less than 1 in the case of $(\mathcal{G}_5, \mathcal{M}_5)$. However, it is noted that $\mathcal{G}_5$ and $\mathcal{M}_5$ are the crisp subsets $\varnothing$ and Z of Z, respectively, and the distance between them should be the maximum distance value 1. The proposed measure $\mathscr{D}$ accurately assigns the correct value of 1 to this pair, demonstrating its ability to handle extreme cases appropriately.

Table 1
Outcome of Example 1.

Measure	$(\mathcal{G}_1, \mathcal{M}_1)$	$(\mathcal{G}_2, \mathcal{M}_2)$	$(\mathcal{G}_3, \mathcal{M}_3)$	$(\mathcal{G}_4, \mathcal{M}_4)$	$(\mathcal{G}_5, \mathcal{M}_5)$	No ties ?
$\mathscr{D}_{NH}$ [10]	0.1000	**0.5000**	**0.5000**	0.2000	1.0000	×
$\mathscr{D}_{NE}$ [10]	0.1000	0.7071	0.5000	0.2000	1.0000	✓
$\mathscr{D}_{NHH}$ [10]	0.1000	**1.0000**	0.5000	0.2000	**1.0000**	×
$\mathscr{D}_{NEH}$ [10]	0.1000	**1.0000**	0.5000	0.2000	**1.0000**	×
$\mathscr{D}_{TG1}$ [11]	0.1000	**0.5774**	0.5000	0.2000	**0.5774**	×
$\mathscr{D}_{TG2}$ [11]	0.1000	**0.5000**	**0.5000**	0.2000	**0.5000**	×
$\mathscr{D}_{TG3}$ [11]	0.1000	**0.5000**	**0.5000**	0.2000	**0.5000**	×
$\mathscr{D}_{TG4}$ [11]	0.0500	**0.2500**	**0.2500**	0.1000	**0.2500**	×
$\mathscr{D}_{TG5}$ [11]	0.1000	**0.5000**	**0.5000**	0.2000	**0.5000**	×
$\mathscr{D}_{TG6}$ [11]	0.0707	0.4082	0.3536	0.1414	0.4082	✓
$\mathscr{D}_{R}$ [12]	0.1000	**0.5000**	**0.5000**	0.2000	1.0000	×
$\mathscr{D}_{L1}$ [13]	0.1000	**1.0000**	0.5000	0.2000	**1.0000**	×
$\mathscr{D}_{L2}$ [13]	0.1000	**0.5000**	**0.5000**	0.2000	1.0000	×
$\mathscr{D}_{G}$ [5]	0.0314	0.4444	0.1667	0.1376	1.0000	✓
$\mathscr{D}_{R1}$ [14]	0.2000	**1.0000**	**1.0000**	0.4000	**1.0000**	×
$\mathscr{D}_{R2}$ [14]	0.3800	**2.0000**	1.5000	0.7200	**2.0000**	×
$\mathscr{D}_{M1}$ [15]	0.0500	0.5000	0.2500	0.1550	1.2500	✓
$\mathscr{D}_{M2}$ [16]	0.0900	**0.2500**	**0.2500**	0.1600	1.0000	×
Proposed $\mathscr{D}$	0.1735	0.5952	0.2381	0.2302	1.0000	✓

The symbol "✓" indicates that the measure produces distinct values for all listed pairs (no ties); "×" indicates at least one tie. $p = 2$ in measure $\mathscr{D}_{TG6}$.

Example 2. This part applies the proposed distance measure to a consumer preference selection problem involving six beverage alternatives. The domain $Z = \{z_1, z_2, z_3, z_4\}$ represents four key attributes of the beverages:

- z_1: Sweetness
- z_2: Price

Table 2
Computational results of various distance measures.

Measure	$(\mathcal{A}_1, \mathcal{S})$	$(\mathcal{A}_2, \mathcal{S})$	$(\mathcal{A}_3, \mathcal{S})$	$(\mathcal{A}_4, \mathcal{S})$	$(\mathcal{A}_5, \mathcal{S})$	$(\mathcal{A}_6, \mathcal{S})$	No ties ?	Result
$\mathscr{D}_{NH}$ [10]	**0.0125**	**0.0125**	0.0257	0.0250	0.0687	0.0437	×	×
$\mathscr{D}_{NE}$ [10]	**0.0354**	**0.0354**	0.0364	0.0707	0.1173	0.0829	×	×
$\mathscr{D}_{NHH}$ [10]	**0.0250**	**0.0250**	0.0530	0.0500	0.1375	0.0875	×	×
$\mathscr{D}_{NEH}$ [10]	**0.0500**	**0.0500**	0.0533	0.1000	0.1677	0.1346	×	×
$\mathscr{D}_{TG1}$ [11]	**0.0289**	**0.0289**	0.0315	0.0577	0.1036	0.0757	×	×
$\mathscr{D}_{TG2}$ [11]	**0.0125**	**0.0125**	0.0257	0.0250	0.0687	0.0437	×	×
$\mathscr{D}_{TG3}$ [11]	**0.0125**	**0.0125**	0.0328	0.0250	0.0875	0.0531	×	×
$\mathscr{D}_{TG4}$ [11]	0.0063	0.0062	0.0128	0.0125	0.0344	0.0219	✓	$\mathcal{A}_2$
$\mathscr{D}_{TG5}$ [11]	**0.0125**	**0.0125**	0.0257	0.0250	0.0687	0.0437	×	×
$\mathscr{D}_{TG6}$ [11]	**0.0204**	**0.0204**	0.0261	0.0408	0.0816	0.0617	×	×
$\mathscr{D}_{R}$ [12]	**0.0125**	**0.0125**	0.0265	0.0250	0.0750	0.0500	×	×
$\mathscr{D}_{L1}$ [13]	**0.1000**	0.0500	0.1805	**0.1000**	0.4500	0.2500	×	×
$\mathscr{D}_{L2}$ [13]	0.0500	0.1000	0.1060	0.2000	0.3000	0.2250	✓	$\mathcal{A}_1$
$\mathscr{D}_{G}$ [5]	0.0119	0.0122	**0.0248**	**0.0248**	0.0652	0.0418	×	$\mathcal{A}_1$
$\mathscr{D}_{R1}$ [14]	**0.0250**	**0.0250**	0.0514	0.0500	0.1375	0.0875	×	×
$\mathscr{D}_{R2}$ [14]	**0.0500**	**0.0500**	0.1021	0.1000	0.2656	0.1681	×	×
$\mathscr{D}_{M1}$ [15]	0.0134	0.0141	0.0273	0.0281	0.0725	0.0453	✓	$\mathcal{A}_1$
$\mathscr{D}_{M2}$ [16]	**0.0119**	**0.0119**	0.0249	0.0225	0.0595	0.0386	×	×
Proposed $\mathscr{D}$	0.0238	0.0149	0.0562	0.0255	0.1027	0.0567	✓	$\mathcal{A}_2$

"No ties" column indicates that the measure produces six strictly distinct values; "✓" indicates that the measure produces distinct values for all listed pairs (no ties); "×" indicates at least one tie. The symbol "×" in the "Result" column indicates that the measure fails to uniquely identify an optimal alternative. $p = 2$ in measure $\mathscr{D}_{TG6}$.

- z_3: Healthiness
- z_4: Packaging appearance

The attribute parameters of the six candidate beverages $\mathcal{A}_1 - \mathcal{A}_6$ are expressed as IVIFSs, and the details are as follows:

$$\mathcal{A}_1 = \left\{ \begin{array}{l} \langle z_1, [0.3000, 0.4000], [0.1000, 0.2000]\rangle, \langle z_2, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \\ \langle z_3, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \langle z_4, [0.2000, 0.3000], [0.1000, 0.2000]\rangle \end{array} \right\},$$

$$\mathcal{A}_2 = \left\{ \begin{array}{l} \langle z_1, [0.2000, 0.3000], [0.2000, 0.3000]\rangle, \langle z_2, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \\ \langle z_4, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \langle z_4, [0.2000, 0.3000], [0.1000, 0.2000]\rangle \end{array} \right\},$$

$$\mathcal{A}_3 = \left\{ \begin{array}{l} \langle z_1, [0.2500, 0.3500], [0.1000, 0.2500]\rangle, \langle z_2, [0.1500, 0.3620], [0.1000, 0.2000]\rangle, \\ \langle z_3, [0.2500, 0.2510], [0.1000, 0.2000]\rangle, \langle z_4, [0.2000, 0.2500], [0.1000, 0.2000]\rangle \end{array} \right\},$$

$$\mathcal{A}_4 = \left\{ \begin{array}{l} \langle z_1, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \langle z_2, [0.2000, 0.3000], [0.3000, 0.4000]\rangle, \\ \langle z_3, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \langle z_4, [0.2000, 0.3000], [0.1000, 0.2000]\rangle \end{array} \right\},$$

$$\mathcal{A}_5 = \left\{ \begin{array}{l} \langle z_1, [0.5000, 0.6000], [0.1000, 0.3000]\rangle, \langle z_2, [0.2000, 0.3000], [0.2000, 0.2000]\rangle, \\ \langle z_3, [0.1500, 0.3500], [0.0500, 0.3000]\rangle, \langle z_4, [0.2000, 0.3000], [0.1000, 0.2500]\rangle \end{array} \right\},$$

$$\mathcal{A}_6 = \left\{ \begin{array}{l} \langle z_1, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \langle z_2, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \\ \langle z_3, [0.2000, 0.3000], [0.2000, 0.2500]\rangle, \langle z_4, [0.4500, 0.4500], [0.1500, 0.3000]\rangle \end{array} \right\}.$$

The ideal point $\mathcal{S}$ of a consumer is represented by an IVIFS:

$$\mathcal{S} = \left\{ \begin{array}{l} \langle z_1, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \langle z_2, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \\ \langle z_3, [0.2000, 0.3000], [0.1000, 0.2000]\rangle, \langle z_4, [0.2000, 0.3000], [0.1000, 0.2000]\rangle \end{array} \right\},$$

which reflects its acceptance range and hesitation degree of each attribute.

The proposed measure $\mathscr{D}$ is employed to compute the distance between each beverage and the consumer's ideal point $\mathcal{S}$. For comparison, 18 representative distance measures for IVIFSs are also applied. The results are summarized in Table 2, and we have the following findings.

(1) The 15 existing distance measures produce identical distance values (entries in bold) for some pairs. Consequently, none of these measures can provide a complete ranking of the six alternatives; each suffers from one tie. However, the proposed measure $\mathscr{D}$ yields six strictly distinct values (0.0238, 0.0149, 0.0562, 0.0255, 0.1027, 0.0567), demonstrating its superior discriminative ability. The symbol "✓" in the "No ties" column indicates that only four measures ($\mathscr{D}_{TG4}$, $\mathscr{D}_{L2}$, $\mathscr{D}_{M1}$, and the proposed $\mathscr{D}$) achieve this property.

(2) Only five measures $\mathscr{D}_{TG4}$, $\mathscr{D}_{L2}$, $\mathscr{D}_{G}$, $\mathscr{D}_{M1}$ and the proposed measure $\mathscr{D}$ are able to identify a single beverage with the smallest distance. $\mathscr{D}_{TG4}$ and the proposed measure $\mathscr{D}$ select $\mathcal{A}_2$, whereas $\mathscr{D}_{L2}$, $\mathscr{D}_{G}$ and $\mathscr{D}_{M1}$ select $\mathcal{A}_1$.

In summary, the numerical evidence presented in Tables 1 and 2 clearly demonstrates that the proposed measure $\mathscr{D}$ avoids any ambiguous equal-distance situations and fully complies with the axiomatic requirements of a distance measure. It thus overcomes the limitations in discriminability and regularity exhibited by several existing distance measures [5, 10–16] for IVIFSs, offering a reliable tool for IVIFS comparisons.

5. Conclusion

In this paper, a new distance measure for IVIFSs is proposed. The measure is based on the min/max operator to fuse the endpoint difference between the membership degree and the non-membership degree interval with two interaction terms, and it is proved that it satisfies the four axiomatic requirements of the distance measure. Then, we also give its simplified form for ordinary IFSs. Finally, the performance of the proposed method is evaluated by comparative experiments. The results show that, unlike the 18 existing IVIFS distance measures, the new measure generates strictly different values for all different IVIFS pairs, fully respects the range of [0,1], and correctly assigns the maximum distance to the complementary crisp set. In the beverage preference decision-making problem, the proposed measure has a unique and clear ranking, and all competing measures have at least one draw. These findings confirm that the proposed distance overcomes the limitations of existing distance measures in terms of discriminability and regularity, and provides a more reliable tool for IVIFS-based decision-making and pattern recognition.

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Conflicts of Interest

The authors declare no conflicts of interest.

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