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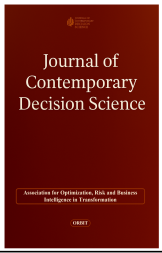

An Intuitionistic Fuzzy Hamming and Euclidean Hesitance Distance Measure for Environmental Sustainability Decision-Making

M. Kaviyarasu[1,*], Nasreen Kausar[2]

[1] Department of mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology Avadi, Chennai, India
[2] Department of Mathematics, Faculty of Arts and Science, Balikesir University, Turkey

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ABSTRACT

Intuitionistic fuzzy distance measures play a fundamental role in modeling uncertainty in multi-criteria decision-making; however, most existing measures do not explicitly exploit hesitation information, which may limit their ability to distinguish alternatives under uncertain conditions. To address this limitation, this study introduces two novel intuitionistic fuzzy distance measures, namely the Euclidean hesitance distance and the Hamming hesitance distance. Unlike conventional approaches that primarily rely on membership and non-membership degrees, the proposed measures explicitly incorporate hesitation, providing a more comprehensive representation of uncertainty and ambiguity. Their mathematical properties are formally established to ensure theoretical consistency and applicability in decision-making environments. To demonstrate their practical usefulness, the proposed measures are applied to an environmental sustainability assessment problem, where evaluations are often characterized by incomplete, vague, and conflicting information. A systematic algorithm is developed to compute and compare the sustainability performance of multiple regions using both distance measures. The application illustrates how incorporating hesitation information enhances the representation of uncertainty and supports a more informative evaluation process. The proposed framework extends the methodological foundations of intuitionistic fuzzy distance measures and provides an effective tool for sustainability assessment and other decision-making problems involving uncertain and ambiguous information.

*Crrsesponding author.
E-mail address: drkaviyarasum@veltech.edu.in

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1. Introduction

Zadeh [1] developed the fuzzy set (FS) theory to handle unpredictability and ambiguity in daily existence and calculating a fuzzy element's significance using membership degrees. Pedrycz [2] presented the fundamentals of fuzzy-set ideas. McBratney & Odeh [3] introduced a method of selecting materials for use in soil also fuzzy measurement-based sciences and fuzzy logic. Kabir and Papadopoulos investigated the use of FSs to ensure security and reliability engineering [4]. To deal with this, Atanassov [5] proposed IFSs, a modification of Zadeh's FS. Likewise [6],which was distinguished by the simultaneous presence of a membership function and a non-membership function. Furthermore, the complexity and unpredictability of the material at hand make it inappropriate for specialists to properly convey their conclusions when dealing with many real-world concerns, it is important to use exact figures. This finding led Atanassov and Gargov [7]. The normalized IF distance measure (IFDM) and the IF similarity measure (IFSM), which are two dual ideas [8], are crucial instruments for evaluating the variations of IFSs for making decisions. As Szmidt et al. [9] shown in terms of the advantages of membership, because of its flexibility in managing ambiguity, IFS provides a method for more human-consistent reasoning. Data mining, pattern identification, image processing, and decision-making have all made use of IFSs.

Consequently, Chen et al.[10] developed many suitable for higher-level double hesitant fuzzy sets (HODHFS) and put forth the idea of HODHFS. those who benefit from situations where ambiguity and uncertainty need a more sophisticated approach to membership and nonmembership. A new DM was defined by Xiao in the IFSs [11] environment. Alkan and Kahraman developed an IF multi-distance metric based on cosine and Euclidean DM [12] developed the application decision-making method, which included the selection of waste disposal destinations. The author introduced a novel IFDM and created a pattern categorization approach based on Gohain et al. IFDM [13] talked about their potential uses in pattern detection, grouping, and decision-making. MADM has made extensive use of IF measures [14]. However, only a small number of distance measurements account for personal preferences in the intricate decision-making process. The concept of nonlinear PFDMs is cited by Dutta et al. [15] discussed how can help with medical diagnostics, pattern recognition, and selecting COVID-19 drugs. Yin et al. introduced an DM framework based on Pythagorean FSs. Ejegwa and Awolola expanded PFDMs by including conventional PFS criteria [16]. IFSs build on classical fuzzy sets by include degrees of membership, non-membership, and reluctance have all made use of these collections [17], used them to solve problems in medical diagnostics and pattern recognition. IFSs extend classical fuzzy sets by include degrees of membership, non-membership, and reluctance. Used the recommended DM Pattern recognition issues. Li et al. [18] introduced novel DMs and similarity measurements for PFSs.

In order to facilitate pattern detection, Hatzimichailidis et al. [19] suggested a distance metric for IFSs. Geometric insights were provided by Yang et al. [20] as investigated consistency between 2D and 3D distances.In order to evaluate credit risk, Shen et al. [21] improved the intuitionistic fuzzy TOPSIS technique by adding a new distance metric. An enhanced similarity metric for IFSs was presented by Song et al. [22], improving accuracy and dependability. By suggesting an improved distance metric for IFSs, this study expands on previous research. The global maximum and lowest variations in membership, non-membership, and hesitation degrees between two IFSs are taken into account by the innovative IFSM that was established by Patel et al. [23]. Using a circular IF VIKOR method, Wang and Chen [24] created a compromise decision-support strategy that addresses MCDM under uncertainty. The complex fuzzy hesitance distance measure (CFHDM) and the complex fuzzy Euclidean hesitance distance measure (CFEHDM), which Khan et al. [25] introduced as new distance measures based on complex fuzzy sets, broaden the current complex fuzzy distance measures by include hesitation values. These metrics were used to tackle difficult decision-making issues, providing a solid approach to managing

complexity in the real world. Thao et al. [26] proposed a novel DMIFSs based on the score function of IF numbers, which addresses the constraints of previous approaches and is applied to pattern recognition, multi-criteria decision-making, and classification problems. Patel et al. [27] developed a new similarity measure for IFSs that successfully captures the degrees. This approach was applied to pattern recognition and multimodal medical image fusion, resulting in increased performance in dealing with uncertainty and integrating several medical imaging modalities. Zhang and Huang [28] created clustering methods based on distance and entropy measures for IFSs, which they used to similarity matrix clustering and demonstrated increased performance in dealing with data uncertainty.
It is clear from the research that is currently available that the majority of distance measures for IFSs concentrate primarily on the membership and non-membership degrees, with the hesitation component being not used. Hesitancy, however, is essential for capturing the underlying ambiguity found in actual expert assessments. This work fills this gap by introducing two new hesitation-based distance measures that explicitly include hesitation degrees in the distance calculations they are HDMIF and the EHDMIF. The mathematical formulation, characteristics, and comparative analysis are thoroughly examined, and their usefulness in making decisions in the real world under uncertainty is illustrated by using a multi-criteria environmental sustainability assessment problem. By developing hesitation distance measure in the framework of IF MCDM, our study closes a significant gap.

1.1 Research Gap and Motivation

Robust distance measures that specifically take hesitation in IFSs into account are lacking in the current literature. Instead of including reluctance into the fundamental framework of the distance calculation, the majority of current models handle it as a secondary or derived value. Furthermore, there aren't many studies that show how these hesitation-based techniques might improve the quality of decisions in real-world settings like healthcare, energy planning, and environmental sustainability. By providing a theoretical and practical framework for hesitancy-aware distance estimates, this research closes this gap. In uncertain and confusing decision-making contexts, effectively recording the level of resemblance or dissimilarity between choices is critical. DMIFSs are important in this context because they account for not just membership and non-membership degrees, but also the intrinsic uncertainties seen in expert judgments. These metrics offer a more refined and accurate method for comparing IF information, allowing decision-makers to make well-informed decisions even when preferences are ambiguous or contradictory. When addressing MCDA problems, which need making decision based on a variety of occasionally conflicting criteria, they are particularly beneficial. By applying fuzzy logic concepts, they also assist managers in describing and managing uncertainty in an efficient manner. Additionally, they offer a variety of potential assessments for every option, which aids decision-makers in communicating uncertainty more effectively and producing more reliable decision results.

1.2 Contribution

Two significant developments in IFDM are presented in this study. First, the two novel distance measures are presented namely, the HDMIF and EHDMIF . By directly including hesitation degrees, these measures deviate from IFDM and enable a more precise representation of doubt and indecision in expert evaluations. Second, the theoretical validity of the suggested models is guaranteed by their mathematical formulation and demonstration of satisfaction with the basic characteristics of distance functions. Additionally, a number of extended forms are developed to improve analytical flexibility in addressing different decision circumstances, such as max, min, and max–min. An application in environmental sustainability assessment, where the decision-making process involves evaluating regions based on pollution levels, climate vulnerability, and economic development potential, is used to demonstrate the efficacy of the suggested approaches and the findings validate the precision and applicability of the suggested techniques by confirming that the economic development potential criterion receives the greatest sustainability score. Finally, A comparison and reduction analysis shows that the suggested measures maintain logical consistency with earlier models when hesitation is zero, reducing to the standard NHDMIFS and NEHDMIFS.

2. Preliminaries

Definition 1 *[1] Let M be a non empty set. A FS T is defined as $T = [t, T_T(t) : t \in M]$. Where $T_T(t) : M \to [0, 1]$ denotes the truth grade of T for any element $t \in M$.*

Definition 2 *[5] Let M be an non empty set.An IFS T is defined as $T = [(t, (T_T(t), F_T(t))) : t \in M]$. Where $T_T(t) : M \to [0, 1]$ and $F_T(t) : M \to [0, 1]$ denotes truth grade and false grade of T for any element $t \in M$, and satisfied the condition $0 \leq (T_T(t) + F_T(t)) \leq 1$. In the grade value of IFS T, the term $m_T(t)$ is said to be an term of membership function, the term $n_T(t)$ is said to be an term of non membership function note that both these functions are real-valued and $m_T(t), n_T(t) \in [0, 1]$. The Hesitance degree of IFS T is denoted by $h_T(t)$ and is given by for membership function,*

$$h_T(x) = |(1 - m_T(x)) - n_T(x))|$$

Definition 3 *[6] Let T_1 and T_2 be two IFS on $T_{T_{1(t)}} = m_{T_{1(t)}}$ and $T_{T_{2(t)}} = m_{T_{2(t)}}$, $F_{T_{1(t)}} = n_{T_{1(t)}}$ and $F_{T_{2(t)}} = n_{T_{2(t)}}$ their grade values. The intersection of T_1 and T_2 is denoted by $T_1 \cap T_2$ and is defined by a function*

$$T_1 \cap T_2 = \{t, min(m_{T_1}(t_1), m_{T_2}(t_2)), max(n_{T_1}(t_1), n_{T_2}(t_2)) : t \in M\}$$

Definition 4 *[6] Let T_1 and T_2 be two IFS on $T_{T_{1(t)}} = m_{T_1}(t)$ and $T_{T_{2(t)}} = m_{T_2}(t)$, $F_{T_{1(t)}} = n_{T_1}(t)$ and $F_{T_{2(t)}} = n_{T_2}(t)$ their grade values. The union of T_1 and T_2 is denoted by $T_1 \cup T_2$ and is defined by a function*

$$T_1 \cup T_2 = \{t, max(m_{T_1}(t_1), m_{T_2}(t_2)), min(n_{T_1}(t_1), n_{T_2}(t_2)) : t \in M\}$$

Definition 5 *[6] Let T be a IFS on M and $T_T(t) = m_T(t)$ and $F_T(t) = n_T(t)$ its grade value. The complement of a IFS T is expressed by T^c and is given by for membership,*

$$T^c = \{t, n_T(t), m_T(t) : t \in M\}$$

Definition 6 *[6] Let T_1 and T_2 be two IFSs and $T_{T_{1(t)}} = m_{T_1}(t)$ and $T_{T_{2(t)}} = m_{T_2}(t)$, $F_{T_{1(t)}} = n_{T_1}(t)$ and $F_{T_{2(t)}} = n_{T_2}(t)$ represent their grade value. The product of T_1 and T_2 is $T_1 \circ T_2$ defined by,*

$$T_1 \circ T_2 = \{t, m_{T_1}(t).m_{T_2}(t), n_{T_1}(t) + n_{T_2}(t) - n_{T_1}(t).n_{T_2}(t) : t \in M\}$$

Definition 7 *[6] Let T_1 and T_2 be two IFSs and $T_{T_{1(t)}} = m_{T_1}(t)$ and $T_{T_{2(t)}} = m_{T_2}(t)$, $F_{T_{1(t)}} = n_{T_1}(t)$ and $F_{T_{2(t)}} = n_{T_2}(t)$ represent their grade value. The probabilistic sum of T_1 and T_2 is $T_1 + T_2$ defined by membership function,*

$$T_1 + T_2 = \{t, m_{T_1}(t) + m_{T_2}(t) - m_{T_1}(t).m_{T_2}(t), n_{T_1}(t).n_{T_2}(t) : t \in M\}$$

3. Intuionistic Fuzzy Distance Measures

The hesitance degree, an intuionistic fuzzy measure for distance that is novel in fuzzy reasoning are commonly accustomed to express the degree of unwillingness or ambiguity about a specific preference. Hesitancy degrees are commonly employed to convey the level of uncertainty or hesitation over a certain preferences or decision. When hesitation values and DMs are taken into account, they frequently pertain to circumstances in which likenesses or preferences are range-based rather than absolute. Hesitancy values can be added to DMs when the individual making the decision is unsure of each criterion's weight or relative relevance. while making judgments involving many criteria. Values can be added to DMs when working with data points that are not exactly part of one cluster but are somewhat part of other clusters. Selecting a measure that accurately depicts the ambiguity or delay in the context of the particular problem or solution is essential. Fuzzy logic approaches or similar methods are commonly employed to address ambiguity and imperfection in the data. When assessing strings or sequences, hesitancy measures are frequently employed to demonstrate the level to match uncertainty. This is especially crucial in cases when there can be ambiguity or insufficient matches, which could lead to inaccurate information retrieval or linkage. In the framework of IFSs, we provide a few special DMs that are referred to as fuzzy DMs with a hesitation degree. Fuzzy and imperfect distance calculations based on intuition Euclidean hesitancy degree measurements in relation to IFSs. Fuzzy and imperfect distance calculations based on intuition Euclidean hesitancy degree measurements with relation to IFSs.

3.1 *Intuitionistic Fuzzy Hesitance Distance Measures*

Definition 8 *[29] HDMIFSs is a function with $d_H : T(M) \times T(M) \to [0,1]$ and $d_H : T(N) \times T(N) \to [0,1]$. For any $T_1, T_2, T_3 \in T(M) \& T(N)$ the following condition should be satisfied.*

i. $0 \leq d_H(T_1, T_2) \leq 1$, $d_H(T_1, T_2) = 0 \Leftrightarrow T_1 = T_2$

ii. $d_H(T_1, T_2) = d_H(T_2, T_1)$

iii. $d_H(T_1, T_3) \leq d_H(T_1, T_2) + d_H(T_2, T_3)$

$$d_H(T_1, T_2) = \frac{[\sum_{n=1}^{i} \mid M_{T_1}(s_i) - M_{T_2}(s_i) \mid + \mid N_{T_1}(s_i) - N_{T_2}(s_i) \mid + \mid h_{T_1}(s_i) - h_{T_2}(s_i) \mid]}{3n} \tag{1}$$

Example 1

$$T_1 = (\frac{0.6, 0.2}{t_1}) + (\frac{0.4, 0.5}{t_2}) + (\frac{0.5, 0, 2}{t_3})$$
$$T_2 = (\frac{0.3, 0.4}{t_1}) + (\frac{0.8, 0.1}{t_2}) + (\frac{0.6, 0.2}{t_3})$$

$$d_H(T_1, T_2) = \frac{\begin{bmatrix} |0.6 - 0.3| + |0.2 - 0.4| + |0.2 - 0.3| \\ +|0.4 - 0.8| + |0.5 - 0.1| + |0.1 - 0.1| \\ +|0.5 - 0.6| + |0.2 - 0.2| + |0.3 - 0.2| \end{bmatrix}}{3n}$$

$$= \frac{[0.3 + 0.2 + 0.1 + 0.4 + 0.4 + 0 + 0.1 + 0 + 0.1]}{3(3)} = 0.177$$

$$d_H(T_1, T_2) = 0.177$$

Theorem 1 *The function defined in Equation (1) is an distance function*

i. First, $d_H(T_1, T_2) \geq 0$ is obviously true.

$$d_H(T_1, T_2) = \frac{[\sum_{n=1}^{i} \mid M_{T_1}(s_i) - M_{T_2}(s_i) \mid + \mid N_{T_1}(s_i) - N_{T_2}(s_i) \mid + \mid h_{T_1}(s_i) - h_{T_2}(s_i) \mid]}{3n}$$
$$= \sum_{n=1}^{i} \frac{[1+1+1]}{3n} = 1$$

$0 \leq d_H(T_1, T_2) \leq 1$

ii.

$$d_H(T_1, T_2) = \frac{\sum_{n=1}^{i}[\mid M_{T_1}(s_i) - M_{T_2}(s_i) \mid + \mid N_{T_1}(s_i) - N_{T_2}(s_i) \mid + \mid h_{T_1}(s_i) - h_{T_2}(s_i) \mid]}{3n}$$
$$= \frac{\sum_{n=1}^{i}[\mid M_{T_2}(s_i) - M_{T_1}(s_i) \mid + \mid N_{T_2}(s_i) - N_{T_1}(s_i) \mid + \mid h_{T_2}(s_i) - h_{T_1}(s_i) \mid]}{3n}$$
$$= d_H(T_2, T_1)$$

iii.

$$d_H(T_1, T_2) = \frac{1}{3n} \sum_{n=1}^{i} [[\mid M_{T_1}(s_i) - M_{T_2}(s_i) \mid + \mid N_{T_1}(s_i) - N_{T_2}(s_i) \mid + \mid h_{T_1}(s_i) - h_{T_2}(s_i) \mid]]$$

$$= \frac{1}{3n} \sum_{n=1}^{i} \begin{bmatrix} [\mid M_{T_1}(s_i) - M_{T_3}(s_i) + M_{T_3}(s_i) - M_{T_2}(s_i) \mid \\ + \mid N_{T_1}(s_i) - N_{T_3}(s_i) + N_{T_3}(s_i) - N_{T_2}(s_i) \mid \\ + \mid h_{T_1}(s_i) - h_{T_3}(s_i) \mid + \mid h_{T_3}(s_i) - h_{T_2}(s_i) \mid] \end{bmatrix}$$

$$= \frac{1}{3n} \sum_{n=1}^{i} \begin{bmatrix} [\mid M_{T_1}(s_i) - M_{T_3}(s_i) \mid \\ + \mid N_{T_1}(s_i) - N_{T_3}(s_i) \mid + \mid h_{T_1}(s_i) - h_{T_3}(s_i) \mid] \end{bmatrix} + \frac{1}{3n} \sum_{n=1}^{i} \begin{bmatrix} [\mid M_{T_2}(s_i) - M_{T_3}(s_i) \mid \\ + \mid N_{T_2}(s_i) - N_{T_3}(s_i) \mid + \mid h_{T_2}(s_i) - h_{T_3}(s_i) \mid] \end{bmatrix}$$

$$= d_H(T_1, T_3) + d_H(T_3, T_2)$$

.

Definition 9 *[29] A EHDMIFSs is a function with $d_{EH} : T(M) \times T(M) \rightarrow [0,1]$ and $d_{EH} : T(N) \times T(N) \rightarrow [0,1]$. For any $T_1, T_2, T_3 \in T(M)$ and $T_1, T_2, T_3 \in T(N)$ the following condition should be satisfied.*

i. $0 \leq d_H(T_1, T_2) \leq 1$,

ii. $d_E H(T_1, T_2) = 0 \Leftrightarrow T_1 = T_2$

iii. $d_E H(T_1, T_2) = d_H(T_2, T_1)$

iv. $d_E H(T_1, T_3) \leq d_H(T_1, T_2) + d_H(T_2, T_3)$

$$d_E H(T_1, T_2) = \sqrt{\frac{\sum_{n=1}^{i}[(M_{T_1}(s_i) - M_{T_2}(s_i))^2 + (N_{T_1}(s_i) - N_{T_2}(s_i))^2 (h_{T_1}(s_i) - h_{T_2}(s_i))^2]}{3n}} \quad (2)$$

Example 2

$$T_1 = (\frac{0.6, 0.2}{t_1}) + (\frac{0.4, 0.5}{t_2}) + (\frac{0.5, 0, 2}{t_3})$$
$$T_2 = (\frac{0.3, 0.4}{t_1}) + (\frac{0.8, 0.1}{t_2}) + (\frac{0.6, 0.2}{t_3})$$

$$d_H\left(T_1, T_2\right) = \sqrt{\frac{\begin{bmatrix} (0.6-0.3)^2 + (0.2-0.4)^2 + (0.2-0.3)^2 \\ +(0.4-0.8)^2 + (0.5-0.1)^2 + (0.1-0.1)^2 \\ +(0.5-0.6)^2 + (0.2-0.2)^2 + (0.3-0.2)^2 \end{bmatrix}}{3n}}$$

$$= \sqrt{\frac{[(0.3)^2 + (0.2)^2 + (0.1)^2 + (0.4)^2 + (0.4)^2 + (0)^2 + (0.1)^2 + (0)^2 + (0.1)^2]}{3(3)}} = 0.0533$$

$$d_H(T_1, T_2) = 0.053$$

Theorem 2 *The function defined in Equation (2) is an distance function.*

i. First, $d_{EH}(T_1, T_2) \geq 0$ is obviously true.
   Next,

$$d_{EH}(T_1, T_2) = \sqrt{\frac{[\sum_{n=1}^{i} \mid M_{T_1}(s_i) - M_{T_2}(s_i) \mid + \mid N_{T_1}(s_i) - N_{T_2}(s_i) \mid + \mid h_{T_1}(s_i) - h_{T_2}(s_i) \mid]}{3n}}$$
$$= \sqrt{\sum_{n=1}^{i} \frac{[1+1+1]}{3n}} = \sqrt{\frac{1}{n}}.$$

   If $n = 1$ then $d_{EH}(T_1, T_2) = 1$ otherwise $d_{EH}(T_1, T_2) < 1$. $0 \leq d_{EH}(T_1, T_2) \leq 1$

ii.

$$d_{EH}(T_1, T_2) = \sqrt{\frac{\sum_{n=1}^{i}[\mid M_{T_1}(s_i) - M_{T_2}(s_i) \mid + \mid N_{T_1}(s_i) - N_{T_2}(s_i) \mid + \mid h_{T_1}(s_i) - h_{T_2}(s_i) \mid]}{3n}}$$
$$= \sqrt{\frac{\sum_{n=1}^{i}[\mid M_{T_2}(s_i) - M_{T_1}(s_i) \mid + \mid N_{T_2}(s_i) - N_{T_1}(s_i) \mid + \mid h_{T_2}(s_i) - h_{T_1}(s_i) \mid]}{3n}}$$
$$= d_{EH}(T_2, T_1)$$

iii.

$$d_{EH}(T_1, T_2) = \sqrt{\frac{1}{3n} \sum_{n=1}^{i} [[\mid M_{T_1}(s_i) - M_{T_2}(s_i) \mid + \mid M_{T_1}(s_i) - M_{T_2}(s_i) \mid + \mid h_{T_1}(s_i) - h_{T_2}(s_i) \mid]]}$$

$$= \sqrt{\frac{1}{3n}\sum_{n=1}^{i}\begin{bmatrix}[\mid M_{T_1}(s_i)-M_{T_3}(s_i)\mid+\mid M_{T_3}(s_i)-M_{T_2}(s_i)\mid\\ +\mid N_{T_1}(s_i)-N_{T_3}(s_i)\mid+\mid N_{T_3}(s_i)-N_{T_2}(s_i)\mid\\ +\mid h_{T_1}(s_i)-h_{T_3}(s_i)\mid+\mid h_{T_3}(s_i)-h_{T_2}(s_i)\mid]\end{bmatrix}}$$

$$= \sqrt{\frac{1}{3n}\sum_{n=1}^{i}\begin{bmatrix}[\mid M_{T_1}(s_i)-M_{T_3}(s_i)\mid\\ +\mid N_{T_1}(s_i)-N_{T_3}(s_i)\mid\\ +\mid h_{T_1}(s_i)-h_{T_3}(s_i)\mid]\end{bmatrix}}+\sqrt{\frac{1}{3n}\sum_{n=1}^{i}\begin{bmatrix}[\mid M_{T_2}(s_i)-M_{T_3}(s_i)\mid\\ +\mid N_{T_2}(s_i)-N_{T_3}(s_i)\mid\\ +\mid h_{T_2}(s_i)-h_{T_3}(s_i)\mid]\end{bmatrix}}$$

$$= d_{EH}(T_1,T_3)+d_{EH}(T_2,T_3).$$

Definition 10 *Let d_H be an HDMIFSs. Then, $(d_H(T_1)^c, d_H(T_2)^c)$ of IFS is*

$$d_H(T_1,T_2)=\frac{\sum_{n=1}^{i}\begin{bmatrix}\mid(1-M_{T_1}(s_i))-(1-M_{T_2}(s_i))\mid\\ +\mid(1-N_{T_1}(s_i))-(1-N_{T_2}(s_i))\mid\\ +\mid(1-h_{T_1}(s_i))-(1-h_{T_2}(s_i))\mid\end{bmatrix}}{3n}$$

Definition 11 *Let d_H be an HDMIFSs. Then the max HDM T_1 andT_2 of two IFS is*

$$d_H(T_1\cup T_2,T_2)=\frac{\sum_{n=1}^{i}\begin{bmatrix}\mid(M_{T_1}(s_i)\cup M_{T_2}(s_i)-M_{T_2}(s_i)\mid\\ +\mid(N_{T_1}(s_i)\cup N_{T_2}(s_i)-N_{T_2}(s_i))\mid\\ +\mid(h_{T_1}(s_i)\cup h_{T_2}(s_i)-h_{T_2}(s_i))\mid\end{bmatrix}}{3n}$$

$$d_H(T_1,T_2\cup T_2)=\frac{\sum_{n=1}^{i}\begin{bmatrix}\mid(M_{T_1}(s_i)-M_{T_2}(s_i)\cup M_{T_2}(s_i))\mid\\ +\mid(N_{T_1}(s_i)-N_{T_2}(s_i)\cup N_{T_2}(s_i))\mid\\ +\mid(h_{T_1}(s_i)-h_{T_2}(s_i)\cup h_{T_2}(s_i))\mid\end{bmatrix}}{3n}$$

where $\cup$ is a max operator.

Definition 12 *Let d_H be an HDMIFSs. Then the min HDM T_1 andT_2 of two IFS is*

$$d_H(T_1\cap T_2,T_2)=\frac{\sum_{n=1}^{i}\begin{bmatrix}\mid(M_{T_1}(s_i)\cap M_{T_2}(s_i))-M_{T_2}(s_i))\mid\\ +\mid(N_{T_1}(s_i)\cap N_{T_2}(s_i)-N_{T_2}(s_i))\mid\\ +\mid(h_{T_1}(s_i)\cap h_{T_2}(s_i)-h_{T_2}(s_i))\mid\end{bmatrix}}{3n}$$

$$d_H(T_1,T_2\cap T_2)=\frac{\sum_{n=1}^{i}\begin{bmatrix}\mid(M_{T_1}(s_i)-(M_{T_2}(s_i)\cap M_{T_2}(s_i)))\mid\\ +\mid(N_{T_1}(s_i)-(N_{T_2}(s_i)\cap N_{T_2}(s_i)))\mid\\ +\mid(h_{T_1}(s_i)-(h_{T_2}(s_i)\cap h_{T_2}(s_i)))\mid\end{bmatrix}}{3n}$$

where $\cap$ is a min operator.

Definition 13 *Let d_H be an HDMIFSs. Then, the max-min HDM T_1 and T_2 of IFS is defined*

$$d_H(T_1\cup T_2,T_1\cap T_2)=\frac{\sum_{n=1}^{i}\begin{bmatrix}\left|(M_{T_1}(s_i)\cup M_{T_2}(s_i))-(M_{T_1}(s_i)\cap M_{T_2}(s_i))\right|\\ +\left|(N_{T_1}(s_i)\cup N_{T_2}(s_i))-(N_{T_1}(s_i)\cap N_{T_2}(s_i))\right|\\ +\left|(h_{T_1}(s_i)\cup h_{T_2}(s_i))-(h_{T_1}(s_i)\cap h_{T_2}(s_i))\right|\end{bmatrix}}{3n}$$

$$d_H(T_1 \cap T_2, T_1 \cup T_2) = \frac{\sum_{n=1}^{i}\begin{bmatrix} \left|(M_{T_1}(s_i) \cap M_{T_2}(s_i)) - (M_{T_1}(s_i) \cup M_{T_2}(s_i))\right| \\ +\left|(N_{T_1}(s_i) \cap N_{T_2}(s_i)) - (N_{T_1}(s_i) \cup N_{T_2}(s_i))\right| \\ +\left|(h_{T_1}(s_i) \cap h_{T_2}(s_i)) - (h_{T_1}(s_i) \cup h_{T_2}(s_i))\right| \end{bmatrix}}{3n}$$

where $\cup$ and $\cap$is a max and min operator.

Definition 14 *Let d_H be an EHDMIFSs. Then, the max HDM T_1 and T_2 of IFS is defined*

$$d_{EH}(T_1 \cup T_2, T_2) = \sqrt{\frac{\sum_{n=1}^{i}\begin{bmatrix} \left((M_{T_1}(s_i) \cup M_{T_2}(s_i)) - M_{T_2}(s_i)\right)^2 \\ +\left((N_{T_1}(s_i) \cup N_{T_2}(s_i)) - N_{T_2}(s_i)\right)^2 \\ +\left((h_{T_1}(s_i) \cup h_{T_2}(s_i)) - h_{T_2}(s_i)\right)^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(T_1, T_2 \cup T_1) = \sqrt{\frac{\sum_{n=1}^{i}\begin{bmatrix} \left(M_{T_1}(s_i) - (M_{T_2}(s_i) \cup M_{T_1}(s_i))\right)^2 \\ +\left(N_{T_1}(s_i) - (N_{T_2}(s_i) \cup N_{T_1}(s_i))\right)^2 \\ +\left(h_{T_1}(s_i) - (h_{T_2}(s_i) \cup h_{T_1}(s_i))\right)^2 \end{bmatrix}}{3n}}$$

Definition 15 *Let d_{EH} be an EHDMIFSs. Then, the max HDM T_1 and T_2 of IFS is defined*

$$d_{EH}(T_1 \cap T_2, T_2) = \sqrt{\frac{\sum_{n=1}^{i}\begin{bmatrix} \left|(M_{T_1}(s_i) \cap M_{T_2}(s_i)) - M_{T_1}(s_i)\right| \\ +\left|(N_{T_1}(s_i) \cap N_{T_2}(s_i)) - N_{T_1}(s_i)\right| \\ +\left|(h_{T_1}(s_i) \cap h_{T_2}(s_i)) - h_{T_2}(s_i)\right| \end{bmatrix}}{3n}}$$

$$d_{EH}(T_1, T_2 \cap T_1) = \sqrt{\frac{\sum_{n=1}^{i}\begin{bmatrix} \left|M_{T_1}(s_i) - (M_{T_2}(s_i) \cap M_{T_1}(s_i))\right| \\ +\left|N_{T_1}(s_i) - (N_{T_2}(s_i) \cap N_{T_1}(s_i))\right| \\ +\left|h_{T_1}(s_i) - (h_{T_2}(s_i) \cap h_{T_1}(s_i))\right| \end{bmatrix}}{3n}}$$

Definition 16 *Let d_{EH} be an EHDMIFSs. Then the max-min HDM T_1 and T_2 of IFS is defined*

$$d_{EH}(T_1 \cup T_2, T_1 \cap T_2) = \sqrt{\frac{\sum_{n=1}^{i}\begin{bmatrix} \left((M_{T_1}(s_i) \cup M_{T_2}(s_i)) - (M_{T_1}(s_i) \cap M_{T_2}(s_i))\right)^2 \\ +\left((N_{T_1}(s_i) \cup N_{T_2}(s_i)) - (N_{T_1}(s_i) \cap N_{T_2}(s_i))\right)^2 \\ +\left((h_{T_1}(s_i) \cup h_{T_2}(s_i)) - (h_{T_1}(s_i) \cap h_{T_2}(s_i))\right)^2 \end{bmatrix}}{3n}}$$

$$d_{EH}(T_1\cap T_2, T_1\cup T_2)=\sqrt{\frac{\sum_{n=1}^{i}\begin{bmatrix}\Big((M_{T_1}(s_i)\cap M_{T_2}(s_i))-(M_{T_1}(s_i)\cup M_{T_2}(s_i))\Big)^2\\+\Big((N_{T_1}(s_i)\cap N_{T_2}(s_i))-(N_{T_1}(s_i)\cup N_{T_2}(s_i))\Big)^2\\+\Big((h_{T_1}(s_i)\cap h_{T_2}(s_i))-(h_{T_1}(s_i)\cup h_{T_2}(s_i))\Big)^2\end{bmatrix}}{3n}}$$

where $\cup$ and $\cap$ is a max and min operator of IFSs.

4. Application For Environmental Sustaiability Using Hamming and Euclidean Hesitence Distance Measure

Multiple criteria, insufficient information, and uncertainty are common components of decision-making issues. A strong foundation for modeling such uncertainty using membership, non-membership, and hesitation degrees is provided by IFSs. In this part, we show how to use the suggested EHDMIF and HDMIF to a decision-making problem pertaining to environmental sustainability. When expert opinions are ambiguous, the application demonstrates the usefulness of the suggested methods in selecting environmentally conscious areas or initiatives.

Table 1
Collection of attributes

E_k	δ_i	b_1	b_2	$\dots$	b_j
3*E_1	δ_1	$(sB_{11}(b_1), sB_{11}(b_1))$	$(sB_{12}(b_2), sB_{12}(b_2)$	$\dots$	$(sB_{1j}(b_j), sB_{1j}(b_j)$
	δ_2	$(sB_{21}(b_1), sB_{21}(b_1)$	$(sB_{22}(b_2), sB_{22}(b_2)$	$\dots$	$(sB_{2j}(b_j), sB_{2j}(b_j)$
	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
	δ_i	$(sB_{i1}(b_1), sB_{i1}(b_1)$	$(sB_{i2}(b_2), sB_{i2}(b_2)$	$\dots$	$(sB_{ij}(b_j), sB_{ij}(b_j)$
3*E_2	δ_1	$(sB_{11}(b_1), sB_{11}(b_1)$	$(sB_{12}(b_2), sB_{12}(b_2)$	$\dots$	$(sB_{1j}(b_j)\ sB_{1j}(b_j)$
	δ_2	$(sB_{21}(b_1), sB_{21}(b_1)$	$(sB_{22}(b_2), sB_{22}(b_2))$	$\dots$	$(sB_{2j}(b_j), sB_{2j}(b_j)$
	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
	δ_i	$(sB_{i1}(b_1), sB_{i1}(b_1)$	$(sB_{i2}(b_2), sB_{i2}(b_2)$	$\dots$	$(sB_{ij}(b_j), sB_{ij}(b_j)$
3*E_k	δ_1	$(sB_{11}(b_1), sB_{11}(b_1)$	$(sB_{12}(b_2), sB_{12}(b_2)$	$\dots$	$(sB_{1j}(b_j), sB_{1j}(b_j)$
	δ_2	$(sB_{21}(b_1), sB_{21}(b_1)$	$(sB_{22}(b_2), sB_{22}(b_2)$	$\dots$	$(sB_{2j}(b_j), sB_{2j}(b_j)$
	$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
	δ_i	$(sB_{i1}(b_1), sB_{i1}(b_1)$	$(sB_{i2}(b_2), sB_{i2}(b_2)$	$\dots$	$(sB_{ij}(b_j), sB_{ij}(b_j)$

The alternative(δ_i) of score function $S(\delta_i)$;i=1,2,...,n and Table 1 lists the values that were assessed in the context of intuitionistic fuzzy sets. $d(\delta_i^{E_j}, \delta_i^{E_j})$ proposed Hesitance or Euclidean DMIFSs.

$$S(\delta_i)=\frac{\sum_{i=1}^{n}(d(\delta_i^{(E_j)}, \delta_i^{(E_k)}))}{n}$$

4.1 Decision Making Algorithm For Proposed Distance Measure

Step 1: Define the decision criteria & assign their grade values. The criteria will be used to evaluate each region, and depending on how well each condition is met, we will assign the grade values.

Step 2: Create an decision matrix. Take three regions (A, B, C). To keep things simple , suppose that the intuionistic fuzzy values are use to evaluate each region based on the three criteria.

Step 3: Determine the distance. Determine the HDMIFSs & EHDMIFSs using equation 1 and 2 and find the distance of each region.

Step 4: Determine each region's overall score. Determine the overall score for each region using score function formula.

Step 5: Rank all the alternatives. After calculating the score value, rank them according to their scores. The above steps are explained in Fig 1

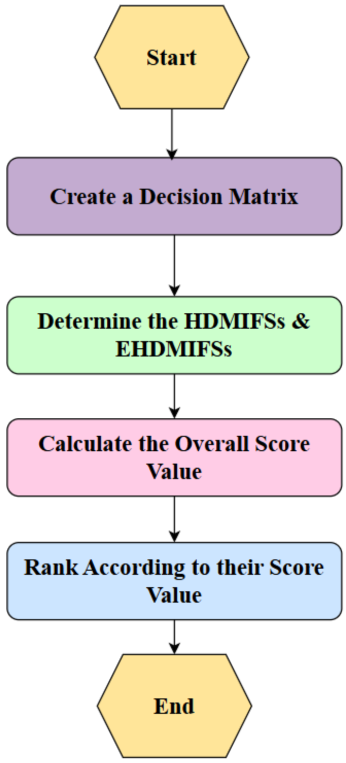

Fig. 1. Steps For Calculating Distance Measure

4.2 Problem Description With Numerical Example

Environmental sustainability is responsible for environment to avoids exhaustion or decrease of natural resources, ensuring that ecosystems remain healthy and productive for future generations. It promotes long-term ecological balance by decreasing pollution, protecting biodiversity, increasing the use of renewable resources, and lowering human activity's environmental impact. In light of escalating global challenges such as climate change, deforestation, water shortages, and air pollution, attaining sustainability has become a vital aim for governments, businesses, and communities. Effective environmental sustainability methods aim not just to safeguard the natural world, but also to promote economic and social growth while maintaining ecological integrity. This example validates the usefulness of the suggested definitions and distance measures by showing how they might be used in actual decision-making situations. The sustainability review committee seeks to select the best region for launching an environmental friendly development program. This selection must be take into consideration by the multiple factors that effect both environmental and economic sustainability. The assessment procedure consists of three major criteria:

1. Pollution level - indicating the environmental quality of each region.
2. Climatic Vulnerability - represents the regions exposure and susceptibility to climate change
3. Economic Development Potential - reflects the prospects for long-term economic growth.

However, due to the ambiguity and vagueness inherent in expert assessments, these criteria are evaluated using both distance measures. Table 2 shows that each region has membership and non-membership degrees for each criterion. Let I_1=Pollution level, I_2= Climatic Vulnerability, I_3=Economic Development Potential. The goal is to use the proposed hesitance-based distance measure to identify which location has the highest level of sustainability among the options. This methodology incorporates environmental, climatic, and economic elements into an IFS, resulting in a more realistic and flexible decision-making process than traditional crisp methods.

Table 2
Evaluated values of alternatives.

	$I_i = (M_i, N_i)$	b_1	b_2	b_3
Region A	I_1	$(0.1, 0.3)$	$(0.3, 0.5)$	$(0.4, 0.2)$
	I_2	$(0.5, 0.4)$	$(0.1, 0.2)$	$(0.3, 0.6)$
	I_3	$(0.7, 0.1)$	$(0.2, 0.8)$	$(0.9, 0)$
Region B	I_1	$(0.6, 0.1)$	$(0.2, 0.6)$	$(0.9, 0)$
	I_2	$(0.2, 0.3)$	$(0.4, 0.5)$	$(0.4, 0.4)$
	I_3	$(0.5, 0.5)$	$(0.5, 0.2)$	$(0.3, 0.3)$
Region C	I_1	$(0.3, 0.2)$	$(0.7, 0.2)$	$(0.9, 0)$
	I_2	$(0.5, 0.3)$	$(0.4, 0.6)$	$(0.1, 0.5)$
	I_3	$(0.7, 0.1)$	$(0.1, 0.8)$	$(0.4, 0.3)$

Intuitionistic Fuzzy Hamming Hesitance Distance Measure

$$d\left(I_1^{(E_1)}, I_1^{(E_2)}\right) = \frac{\begin{bmatrix} |0.1-0.6| + |0.3-0.2| + |0.4-0.9| \\ +|0.3-0.1| + |0.5-0.6| + |0.2-0| \\ +|0.6-0.3| + |0.2-0.2| + |0.4-0.1| \end{bmatrix}}{3(3)}$$

$$= \frac{[0.5+0.1+0.5+0.3+0+0.3+0.2+0.1+0.2]}{9} = 0.24$$

$$d\left(I_1^{(E_1)}, I_1^{(E_3)}\right) = \frac{\begin{bmatrix} |0.1-0.3|+|0.3-0.7|+|0.4-0.9| \\ +|0.3-0.2|+|0.5-0.2|+|0.2-0| \\ +|0.6-0.5|+|0.2-0.1|+|0.4-0.1| \end{bmatrix}}{3(3)}$$

$$= \frac{[0.2+0.4+0.5+0.1+0.3+0.2+0.1+0.1+0.3]}{9} = 0.24$$

$$d\left(I_1^{(E_2)}, I_1^{(E_3)}\right) = \frac{\begin{bmatrix} |0.6-0.3|+|0.2-0.7|+|0.9-0.9| \\ +|0.1-0.2|+|0.6-0.2|+|0-0| \\ +|0.3-0.5|+|0.2-0.1|+|0.1-0.1| \end{bmatrix}}{3(3)}$$

$$= \frac{[0.3+0.5+0+0.1+0.4+0+0.2+0.1+0]}{9} = 0.17$$

$$\begin{array}{ll} d(I_2^{(E_1)}, I_2^{(E_2)}) = 0.26, & d(I_2^{(E_1)}, I_2^{(E_3)}) = 0.24, \\ d(I_2^{(E_2)}, I_2^{(E_3)}) = 0.16, & d(I_3^{(E_1)}, I_3^{(E_2)}) = 0.35, \\ d(I_3^{(E_1)}, I_3^{(E_3)}) = 0.14, & d(I_3^{(E_2)}, I_3^{(E_3)}) = 0.22, \end{array}$$

Using Definition **??**, we will get the decision matrix as

$$[DM]_{3\times 3} = \begin{bmatrix} 0.24 & 0.24 & 0.17 \\ 0.26 & 0.24 & 0.15 \\ 0.35 & 0.14 & 0.22 \end{bmatrix}$$

$$S(L_i) = \frac{\sum_{i=1}^{3}(d(I_i^{(E_j)}, I_i^{(E_j)})}{3}$$

$$S(L_1) = \left(\sum_{i=1}^{3} \frac{(0.24+0.24+0.17)}{3}\right) = 0.21$$

$$S(L_2) = \left(\sum_{i=1}^{3} \frac{(0.26+0.24+0.16)}{3}\right) = 0.22$$

$$S(L_3) = \left(\sum_{i=1}^{3} \frac{(0.35+0.14+0.22)}{3}\right) = 0.23$$

$$0.21 < 0.22 < 0.23$$

$$S(L_1) < S(L_2) < S(L_3)$$

$S(L_3) =$ Economic development potential is thus selected.

Intuitionistic Fuzzy Euclidean Hesitance Distance Measure

$$d\left(I_1^{(E_1)}, I_1^{(E_2)}\right) = \sqrt{\frac{\begin{bmatrix} (0.1-0.6)^2+(0.3-0.2)^2+(0.4-0.9)^2 \\ +(0.3-0.1)^2+(0.5-0.6)^2+(0.2-0)^2 \\ +(0.6-0.3)^2+(0.2-0.2)^2+(0.4-0.1)^2 \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{[(0.5)^2+(0.1)^2+(0.5)^2+(0.3)^2+(0)^2+(0.3)^2+(0.2)^2+(0.1)^2+(0.2)^2]}{9}} = 0.28$$

$$d\left(I_1^{(E_1)}, I_1^{(E_3)}\right) = \sqrt{\frac{\begin{bmatrix} (0.1-0.3)^2+(0.3-0.7)^2+(0.4-0.9)^2 \\ +(0.3-0.2)^2+(0.5-0.2)^2+(0.2-0)^2 \\ +(0.6-0.5)^2+(0.2-0.1)^2+(0.4-0.1)^2 \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{[(0.2)^2+(0.4)^2+(0.5)^2+(0.1)^2+(0.3)^2+(0.2)^2+(0.1)^2+(0.1)^2+(0.3)^2]}{9}} = 0.26$$

$$d\left(I_1^{(E_2)}, I_1^{(E_3)}\right) = \sqrt{\frac{\begin{bmatrix} (0.6-0.3)^2+(0.2-0.7)^2+(0.9-0.9)^2 \\ +(0.1-0.2)^2+(0.6-0.2)^2+(0-0)^2 \\ +(0.3-0.5)^2+(0.2-0.1)^2+(0.1-0.1)^2 \end{bmatrix}}{3(3)}}$$

$$= \sqrt{\frac{[(0.3)^2+(0.5)^2+(0)^2+(0.1)^2+(0.4)^2+(0)^2+(0.2)^2+(0.1)^2+(0)^2]}{9}} = 0.24$$

$$\begin{aligned} d(I_2^{(E_1)}, I_2^{(E_2)}) &= 0.30, \quad d(I_2^{(E_1)}, I_2^{(E_3)}) = 0.31, \\ d(I_2^{(E_2)}, I_2^{(E_3)}) &= 0.19, \quad d(I_3^{(E_1)}, I_3^{(E_2)}) = 0.38, \\ d(I_3^{(E_1)}, I_3^{(E_3)}) &= 0.21, \quad d(I_3^{(E_2)}, I_3^{(E_3)}) = 0.29, \end{aligned}$$

Using Definition **??**, we will get the decision matrix as

$$[DM]_{3\times 3} = \begin{bmatrix} 0.28 & 0.26 & 0.24 \\ 0.30 & 0.31 & 0.19 \\ 0.38 & 0.21 & 0.29 \end{bmatrix}$$

$$\begin{aligned} S(L_i) &= \frac{\sum_{i=1}^{3}(d(I_i^{(E_j)}, I_i^{(E_j)})}{3} \\ S(L_1) &= \left(\sum_{i=1}^{3} \frac{(0.28+0.26+0.24)}{3}\right) = 0.26 \\ S(L_2) &= \left(\sum_{i=1}^{3} \frac{(0.30+0.31+0.19)}{3}\right) = 0.27 \\ S(L_3) &= \left(\sum_{i=1}^{3} \frac{(0.38+0.21+0.29)}{3}\right) = 0.29 \\ &0.26 < 0.27 < 0.29 \\ &S(L_1) < S(L_2) < S(L_3) \end{aligned}$$

$S(L_3)$ = Economic development potential is thus selected, by the score functions based on EHD-MIFSs. The graphical representation is also given in Figure2. So, By both the cases HDMIFSs and EHDMIFSs, economic development potential has to be first taken into account for environmental sustainability.

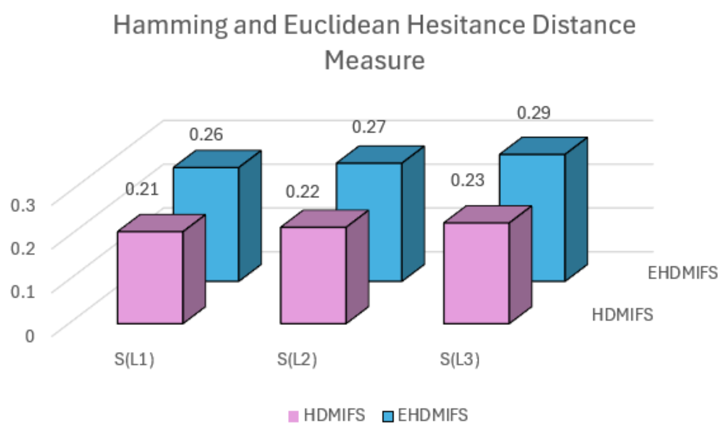

Fig. 2. Hamming and Euclidean Hesitance Distance Measure

4.3 *Comparative Analysis For Normalized Distance Measures*

This section compares the existing NHDMIFS and NEDMIFS with the proposed HDMIFS and EDMIFS. The suggested HDMIFS and EDMIFS can be reduced to NHDMIFS and NEDMIFS, respectively, with specific parameter limitations. Remarks 1 and 2 show the theoretical coherence of these method. We also describe decision-making strategies that do not take hesitation degrees into account and were created to utilizing more intricate fuzzy distance measurements. Below is a summary of the overall comparison evaluation.

Remark 1 *If we assume that the hesitancy degree as zero, then the HDMIFS is reduced to NHDMIFS.*

$$d_H(T_1,T_2)=\frac{[\sum_{n=1}^{i}\mid M_{T_1}(s_i)-M_{T_2}(s_i)\mid+\mid N_{T_1}(s_i)-N_{T_2}(s_i)\mid]}{2n}$$

Remark 2 *If we assume that the hesitancy degree as zero, then the HDMIFS is reduced to NHDMIFS.*

$$d_EH(T_1,T_2)=\sqrt{\frac{\sum_{n=1}^{i}[(M_{T_1}(s_i)-M_{T_2}(s_i))^2+(N_{T_1}(s_i)-N_{T_2}(s_i))^2]}{2n}}$$

The NHDMIFS between the values assessed by several experts corresponding to alternatives can be used to address the aforementioned decision-making situation given in Table 1.

Intuitionistic Fuzzy Normalized Hamming Distance Measure

$$d\left(I_1^{(E_1)},I_1^{(E_2)}\right)=\frac{\begin{bmatrix}|0.1-0.6|+|0.3-0.2|+|0.4-0.9|\\+|0.3-0.1|+|0.5-0.6|+|0.2-0|\end{bmatrix}}{2(3)}$$

$$=\frac{[0.5+0.1+0.5+0.2+0.1+0.2]}{6}=0.26$$

$$d\left(I_1^{(E_1)},I_1^{(E_3)}\right)=\frac{\begin{bmatrix}|0.1-0.3|+|0.3-0.7|+|0.4-0.9|\\+|0.3-0.2|+|0.5-0.2|+|0.2-0|\end{bmatrix}}{3(3)}$$

$$=\frac{[0.2+0.4+0.5+0.1+0.3+0.2]}{6}=0.28$$

$$d\left(I_1^{(E_2)}, I_1^{(E_3)}\right) = \frac{\begin{bmatrix} |0.6-0.3|+|0.2-0.7|+|0.9-0.9| \\ +|0.1-0.2|+|0.6-0.2|+|0-0| \end{bmatrix}}{2(3)}$$

$$= \frac{[0.3+0.5+0+0.1+0.4+0+]}{6} = 0.21$$

$$\begin{array}{ll} d(I_2^{(E_1)}, I_2^{(E_2)}) = 0.21, & d(I_2^{(E_1)}, I_2^{(E_3)}) = 0.18, \\ d(I_2^{(E_2)}, I_2^{(E_3)}) = 0.13, & d(I_3^{(E_1)}, I_3^{(E_2)}) = 0.4, \\ d(I_3^{(E_1)}, I_3^{(E_3)}) = 0.15, & d(I_3^{(E_2)}, I_3^{(E_3)}) = 0.28, \end{array}$$

Using Definition **??**, we will get the decision matrix as

$$[DM]_{3\times 3} = \begin{bmatrix} 0.26 & 0.28 & 0.21 \\ 0.21 & 0.18 & 0.13 \\ 0.4 & 0.15 & 0.28 \end{bmatrix}$$

$$S(L_i) = \frac{\sum_{i=1}^{3}(d(I_i^{(E_j)}, I_i^{(E_j)})}{3}$$

$$S(L_1) = \left(\sum_{i=1}^{3} \frac{(0.26+0.28+0.21)}{3}\right) = 0.25$$

$$S(L_2) = \left(\sum_{i=1}^{3} \frac{(0.21+0.18+0.13)}{3}\right) = 0.17$$

$$S(L_3) = \left(\sum_{i=1}^{3} \frac{(0.4+0.15+0.28)}{3}\right) = 0.27$$

$$0.17 < 0.25 < 0.27$$

$$S(L_2) < S(L_1) < S(L_3)$$

$S(L_3) =$ Economic development potential is thus selected.
Intuitionistic Fuzzy Normalized Euclidean Distance Measure

$$d\left(I_1^{(E_1)}, I_1^{(E_2)}\right) = \sqrt{\frac{\begin{bmatrix} (0.1-0.6)^2+(0.3-0.2)^2+(0.4-0.9)^2 \\ +(0.3-0.1)^2+(0.5-0.6)^2+(0.2-0)^2 \end{bmatrix}}{2(3)}}$$

$$= \sqrt{\frac{[(0.5)^2+(0.1)^2+(0.5)^2+(0.2)^2+(0.1)^2+(0.2)^2]}{6}} = 0.31$$

$$d\left(I_1^{(E_1)}, I_1^{(E_3)}\right) = \sqrt{\frac{\begin{bmatrix} (0.1-0.3)^2+(0.3-0.7)^2+(0.4-0.9)^2 \\ +(0.3-0.2)^2+(0.5-0.2)^2+(0.2-0)^2 \end{bmatrix}}{2(3)}}$$

$$= \sqrt{\frac{[(0.2)^2+(0.4)^2+(0.5)^2+(0.1)^2+(0.3)^2+(0.2)^2]}{6}} = 0.31$$

$$d\left(I_1^{(E_2)}, I_1^{(E_3)}\right) = \sqrt{\frac{\begin{bmatrix} (0.6-0.3)^2 + (0.2-0.7)^2 + (0.9-0.9)^2 \\ +(0.1-0.2)^2 + (0.6-0.2)^2 + (0-0)^2 \end{bmatrix}}{2(3)}}$$

$$= \sqrt{\frac{[(0.3)^2 + (0.5)^2 + (0)^2 + (0.1)^2 + (0.4)^2 + (0)^2]}{6}} = 0.29$$

$$\begin{aligned} d(I_2^{(E_1)}, I_2^{(E_2)}) &= 0.23, \quad d(I_2^{(E_1)}, I_2^{(E_3)}) = 0.22, \\ d(I_2^{(E_2)}, I_2^{(E_3)}) &= 0.18, \quad d(I_3^{(E_1)}, I_3^{(E_2)}) = 0.42, \\ d(I_3^{(E_1)}, I_3^{(E_3)}) &= 0.24, \quad d(I_3^{(E_2)}, I_3^{(E_3)}) = 0.34, \end{aligned}$$

Using Definition **??**, we will get the decision matrix as

$$[DM]_{3\times 3} = \begin{bmatrix} 0.31 & 0.31 & 0.29 \\ 0.23 & 0.22 & 0.18 \\ 0.42 & 0.24 & 0.34 \end{bmatrix}$$

$$\begin{aligned} S(L_i) &= \frac{\sum_{i=1}^{3}(d(I_i^{(E_j)}, I_i^{(E_j)})}{3} \\ S(L_1) &= \left(\sum_{i=1}^{3} \frac{(0.31+0.31+0.29)}{3}\right) = 0.30 \\ S(L_2) &= \left(\sum_{i=1}^{3} \frac{(0.23+0.22+0.18)}{3}\right) = 0.21 \\ S(L_3) &= \left(\sum_{i=1}^{3} \frac{(0.42+0.24+0.34)}{3}\right) = 0.33 \\ & 0.21 < 0.30 < 0.33 \\ & S(L_2) < S(L_1) < S(L_3) \end{aligned}$$

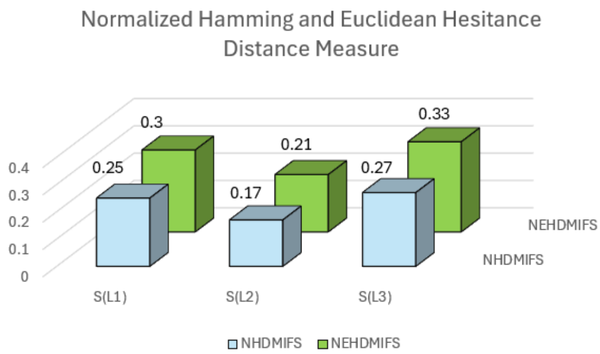

Fig. 3. Normalized Hamming and Euclidean Hamming Distance Measure

$S(L_3)$ = Economic development potential is thus selected, by the score functions based on NEHDMIFSs.The graphical representation is also given in Figure3.

The newly introduced models not only maintain theoretical consistency but also provide better performance in managing uncertainty, according to a comparison between the proposed HDMIF and EHDMIF and the current NHDMIFS and NEDMIFS. Both HDMIF and EHDMIF reduce to their normalized equivalents when the hesitation degree is taken to be zero, indicating the suggested framework's logical and a capacity for generalization. The suggested models, shows increased sensitivity to differences in expert opinions when hesitation is taken into account, producing rankings that are more accurate and realistic. Decision-makers may now more accurately represent the actual uncertainty present in IF evaluations. In contrast to the traditional normalized distance models, the suggested HDMIF and EHDMIF offer a more comprehensive and human-consistent representation of uncertainty.

4.4 *Sensitivity Analysis Based on Distance Measures*

A sensitivity analysis was conducted by comparing the outcomes of the suggested HDMIFS and EHDMIFS with the current NHDMIFS and NEHDMIFS in order to assess the robustness of the suggested distance measures. The aim is to determine whether different formulations of the distance function lead to consistent decision outcomes.

Table 3
Finding the best optimal

Distance Measures	Score Value	Ranking values	Best optimal
$HDMIFS$	$0.21 < 0.22 < 0.23$	$S(L_1) < S(L_2) < S(L_3)$	$S(L_3)$
$EHDMIFS$	$0.26 < 0.27 < 0.29$	$S(L_1) < S(L_2) < S(L_3)$	$S(L_3)$
$NHDMIFS$	$0.17 < 0.25 < 0.27$	$S(L_2) < S(L_1) < S(L_3)$	$S(L_3)$
$NEHDMIFS$	$0.21 < 0.30 < 0.33$	$S(L_2) < S(L_1) < S(L_3)$	$S(L_3)$

Table 3 demonstrates that the options ranking order is constant across all distance metrics, with $S(L_3)$ being the best and most optimal option. This consistency demonstrates that, independent of the precise form of the distance computation, the suggested distance measure generate stable and trustworthy decisions. As a result, the decision-making framework demonstrates significant model robustness, which is an essential characteristic for practical sustainability assessments in the face of uncertainty.

4.5 *Advantages of Hesitance Distance Measures of Intuitionistic Fuzzy Sets*

IF's hesitancy values and distance measure offer a solid basis for handling and modeling decision-making ambiguity. These metrics provide a flexible and trustworthy way to represent ambiguous preferences, assisting decision-makers in making more confident and informed decisions. Decision-makers can assess their lack of confidence or unwillingness toward various possibilities directly by using hesitation values in fuzzy distance measurements. Managers can provide a more accurate depiction of ambiguity by giving fuzzy set objects values that reflect their level of doubt regarding membership. Furthermore, intuitionistic fuzzy distance measures with hesitancy values provide an adaptable framework of conveying desires that are not quite clear. By displaying different levels of uncertainty for many different factors, decision-makers may illustrate the nature of unresolved when deciding on situations.

5. Conclusion

In this paper, two novel distance measures HDMIF and the EHDMIF for IFSs were developed to effectively capture the uncertainty and hesitation in decision-making situation. In contrast to conventional fuzzy methods, the suggested measures offer a more accurate representation of ambiguity by incorporating hesitation degrees into the similarity calculation. Both measures consistently identified the same most sustainable criterion, validating their robustness. These hesitation based metrics improve the accuracy, adaptability, and interpretability of MCDM processes, as shown by their application to environmental sustainability issues. Future research can concentrate on applying these metrics to more general fuzzy frameworks, like q-rung orthopair fuzzy sets, Pythagorean fuzzy sets, or Fermatean fuzzy sets, and integrating them into dynamic settings and collective decision-making. Furthermore, incorporating optimization-based weighting techniques or machine learning could enhance performance and adaptability in intricate real-world applications.

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Conflicts of Interest

The authors declare no conflicts of interest.

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