



A Novel MADM Method based on Grey Preference Degree and its Application in Supplier Selection

Supratim Mukherjee^{1,*}, Amitava Chatterjee²

¹ Department of Mathematics, Government General Degree College, Tehatta, West Bengal, India

² Department of Mathematics, J.S.S.R.K. High School, Purba Bardhaman, West Bengal, India

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ABSTRACT

Solving Multi-Attribute Decision-Making (MADM) problems under uncertainty is a challenging task, particularly in group decision-making environments where decision-makers may differ in their experience, expertise, and evaluation capabilities. Therefore, determining the appropriate weights of decision-makers is an important aspect of the decision-making process. This study proposes a three-phase group decision-making framework that addresses uncertainty using Interval Grey Numbers (IGNs). In the first phase, the weights of decision-makers are determined based on the deviations between their individual evaluations and predefined ideal decisions. In the second phase, a novel comparison measure, called the Grey Preference Degree, is introduced to compare IGNs and address the limitations of existing comparison approaches. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is then employed to evaluate and rank the alternatives. In the third phase, a classification procedure is developed to group alternatives according to their performance, supporting future decision-making activities. The proposed methodology is demonstrated through a practical supplier selection problem. A comparative analysis is conducted to examine its methodological advantages over existing approaches. The proposed framework integrates decision-maker weighting, alternative ranking, and classification into a unified decision-making process under uncertainty.

1. Introduction

Multi Attribute Decision Making has been a topic of extensive research in current years. Before the introduction of Fuzzy Logic by Zadeh [1], traditional statistical approaches were applied to solve MADM problems, even when, the situation was full of uncertain elements. But, after the development of uncertainty-based logics and operational tools, new methodologies based on these approaches have been proposed frequently. Different types of MADM problem models are there in the literature considering the nature of the uncertain environment. Normally a MADM problem consists of three components: alternatives (persons/companies/institutions etc which are to be assessed), attributes (criteria on the basis of which the decision-making process will progress),

* Corresponding author.

E-mail address: author.mail@gmail.com

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experts or decision makers (individuals whose decisions on the alternatives with respect to the attributes are to be processed). To obtain the ranking of the alternatives, different researchers have proposed a lot of methodologies in the literature.

Several quantitative techniques have been used for MADM problems in literature. Some commonly used methods include the Analytical Hierarchy Process (AHP), Analytical Network Process (ANP), Technique for Ordered Preference by Similarity to Ideal Solution (TOPSIS), Data Envelopment Analysis (DEA), and various weighting techniques. Supplier selection is a key aspect of Supply Chain Management (SCM). In today's highly competitive business environment, producing high-quality and cost-effective products is challenging without a reliable supplier or a group of capable suppliers. A well-chosen supplier helps lower procurement costs and strengthens a company's competitive position. Therefore, the primary goal of supplier evaluation is to maximize overall value for the purchaser while fostering strong relationships between buyers and suppliers.

Studies show that the use of AHP improves decision-making efficiency and shortens the time needed to identify the best alternative. Many authors have used AHP in classical and in modified form with fuzzy set and others in their literature. Akarte *et al.*, [2] have used a web-based AHP by considering 18 criteria to choose alternatives in an automobile casting. A five step AHP- based model have proposed by Muralidharan *et al.*, [3] to assist decision-makers in rating and selecting alternatives by considering 9 criteria in a bicycle manufacturing company. Chan [4] has used an interactive selection model with AHP to improve decision makers in selecting the alternatives. Similar work was done by Liu and Hai [5]. A decision-making problem containing 14 criteria based on AHP was developed and a sensitivity analysis was performed by Chan and Kumar [6] in their literature. A fuzzy AHP was used by Kaharaman *et al.*, [7] to select the best supplier by using linguistic variable for supplier's criteria. Chan and Kumar [6] have used fuzzy AHP and the fuzzy synthetic extent analysis method in their literature. Bottani and Rizzi [8] applied fuzzy logic to develop an integrated approach that combines cluster analysis and AHP for grouping and ranking alternatives in the supplier selection process. Sarkis and Talluri [9] have used by ANP method for alternative selection process considering the internal interdependency between alternative evaluating factors. An ANP model was implemented by Bayazit [10] and Gencer and Gürpınar [11] to tackle supplier selection problem. The DEA model was introduced by Liu *et al.*, [12] to select the best supplier having greater supply variety. A suggestion to use DEA and AHP together to evaluate performance of suppliers was proposed by Ramanathan [13]. An integrated AHP-DEA model was also used by Sevkli *et al.*, [14]. TOPSIS is based on a very simple principle. Here, under the framework of MADM model, the evaluator has to build an ideal solution which stands best among the given alternatives with respect to all the attributes. The TOPSIS method for handling multiple attribute decision-making problems was presented by Hwang and Yoon [15]. Boran *et al.*, [16] developed a three-step multi-criteria model using fuzzy TOPSIS to evaluate the environmental performance of suppliers. Also Awasthi *et al.*, [17] anticipated the combination of TOPSIS method with intuitionistic fuzzy set for choosing a right supplier in group decision making procedure. To handle the group decision making problems, an extended TOPSIS method was proposed by chen [18]. Other extended work on TOPSIS method was done by Xie and Xin [19], Sreekumar *et al.*, [20], Mukherjee and Kar [21] and many other researchers. More detailed review can be found in [22-24].

The assessment of the attributes values plays an important role in MADM and especially in supplier selection process. In the decision-making process, due to the increasing complexity of the socio-economic environment and the vagueness of inherent subjective nature of human thinking the information about the attribute values is usually uncertain in nature. To tackle this type of uncertainty fuzzy set theory [1] is very much popular among many authors. In this literature, Grey System Theory [25] is applied to handle uncertainties, as it focuses on addressing uncertainty problems in small

samples and limited information—challenges that traditional probability and fuzzy mathematics struggle to resolve. One notable characteristic of Grey System Theory is its ability to construct models with minimal data. The key difference between fuzzy mathematics and Grey System Theory lies in their approach to the properties of intensities and extensions of the studied objects [26]. Grey System Theory primarily investigates objects with a well-defined extension but an unclear intension. Moreover, grey system theory considers the condition of the fuzziness. That is, it can flexibly deal with the fuzziness situation [27–29]. In the literature we can find many application of grey system theory. Li *et al.*, [27] proposed a possibility degree among the suppliers in grey system theory and then advancement of the method of Li *et al.*, [27] was proposed by Jadidi *et al.*, [30]. Grey system theory was proposed in stock market prediction by Kanyan *et al.*, [31]. Tseng [32] have applied grey system theory for assessment of estate agents based on quality services. In the field of financial performance of container transport grey system theory was successfully applied by Wang [33]. For forecasting integrated circuit output grey theory was applied by Li Chang and Chao Huang [34]. Grey system theory was used for evaluation of airports according to the quality of services, optimization and demand forecasting by Kose *et al.*, [35]. Özdemir and Deste [36], Sanayei *et al.*, [37] and Hasheme *et al.*, [38] have applied grey system theory in supplier selection process. In group multi criteria decision making methods grey system theory model was introduced by Zhang and Liu [39]. For the measurement of performance in insurance sector grey system theory was introduced by Peker and Baki [40]. Grey system theory was successfully applied by Jin *et al.*, [41] for analyzing the credit risk from the perspective risk management using grey incidence analysis. In green supply management grey system theory was applied by Bali *et al.*, [42] and Datta *et al.*, [43]. In their literature Zavadshas *et al.*, [44] presented a weighted aggregated sum product assessment with grey attribute score. Nyaoga *et al.*, [45] developed an approach to integrate the advantage of grey system theory and TOPSIS to evaluate and rank value chain performance. In the literature of Zaher *et al.*, [46], they proposed two grey based methods for multi-objective optimization. In his literature, analysis of the possibility of grey based TOPSIS method to access the occupational risk management was discussed by Tabor [47]. A hybrid Multi criteria decision making technique using grey fuzzy TOPSIS was proposed by Feng *et al.*, [48].

The complexity of socio-economic environment is increasing, and it makes the situation less possible for single decision maker (DM) to judge all related aspects of a problem. As a result, many decision making process, in the real world consider a group of decision makers [49]. Shifting from single decision maker to a group of decision makers introduces an effective deal of complexity into the analysis [50]. Each decision maker's knowledge structure, skill, judgment level, experience and personal preference are different either they are from the same field or from different fields. That is, the weights of the decision makers are different. It is obvious that the weights of the DMs (experts) influence the process of decision making in a group decision making environment. But, in reality, it is almost impossible to arrange a homogeneous group of Decision Makers whose decision-making skills (experience, knowledge etc.) are the same. In MADM, as the complexity of the socio-economic environment increase, the non-homogeneous group of experts with equally imposed weights produce more complexity. So, in order to obtain a satisfactory result, the weights of the experts should be considered.

There are many studies on the weights of decision makers. Chen and Yang [51] in their literature proposed a method for finding the weight of the decision makers in which the weight of the decision makers is calculated according to the proximity of the evaluation value of each decision maker to the average evaluation value of the group. To find the weight of the decision makers Yue [52] extended the TOPSIS method with interval numbers. A linear programming model based on a deviation function was proposed by Abootalebi *et al.*, [53] to find the optimal expert weights. Projection rule

was used to determine the weight of decision makers by Yue [54]. Uniform consistency-based study from complete linguistic preference relation for the weight of the decision makers was done by Cheng *et al.*, [55]. The distance between the decision matrices formed by the decisions of individual decision makers is used for the weights of the decision makers in the literature of Meng *et al.*, [56]. A similarity-based method was proposed by Wan and Dong [57] for the determination of the weight of the decision makers. Also, Eshlaghy and Farokhi [58] have developed proposed a method to measure the importance and weights of the Decision Makers using Eigen vector method, which is based on pair wise comparison technique.

The main factor of each MADM and in specially supplier selection process is the preferences determined by the decision makers for the alternatives concerning each criterion and so the method behind the ranking of the alternatives is equally important. In this context the proposed concept in this literature is very relevant. From the discussion of existing literatures, some facts are prominent and they are, in some literature weight of the decision makers are not considered, the process of degreying (whitening) the grey numbers has been implemented too early in the methodology, grey numbers are compared by distance only etc. and these will affect the whole selection procedure. Also, in most of the literature using grey numbers, possibility measure proposed by Li *et al.*, [27] has been applied in the process of comparison but that method proposed by Li *et al.*, [27] have some drawbacks and that are addressed in our literature here. In this context a new measure named preference degree has been introduced in this literature and the drawbacks, discussed earlier have been corrected. Not only that, classification of the suppliers in different categories has also been done here to help of the authority to choose the alternatives. The aim of this literature is to present a suitable method for MADM process by introducing a new measure of preference between two interval grey numbers and by classification of the suppliers into different groups. This study attempts to present a practical approach that will be more compatible with the real world.

The organization of the current study has been as follows: Section 2 sums up the preliminary concepts related to the method. In section 3, a detailed review of some existing grey based MADM approaches has been done for further comparative study. In Section 4, a new method on comparing Interval Grey Numbers is introduced. In section 5, the phases of the methodology are demonstrated in detail. Section 6 deals with a case study where the problem of selecting a suitable supplier is solved using the methodology. A necessary comparative study is discussed in Section 7. Section 8 concludes the whole.

2. Preliminaries

Grey systems are defined as systems that incorporate uncertainty, represented by grey numbers and grey variables.

Definition 1. Let X be the universal set. Then a grey system G of X is defined by the two mappings $\bar{\mu}_G(x)$ and $\underline{\mu}_G(x)$ where $\bar{\mu}_G(x): x \rightarrow [0, 1]$, $\underline{\mu}_G(x): x \rightarrow [0, 1]$, $\bar{\mu}_G(x) \geq \underline{\mu}_G(x)$, $x \in X$, $X = \mathbb{R}$.

$\bar{\mu}_G(x)$ and $\underline{\mu}_G(x)$ represents the upper and lower membership functions in G respectively.

Note: When $\bar{\mu}_G(x) = \underline{\mu}_G(x)$, the grey set G becomes a Fuzzy set. Grey Theory, therefore, considers the presence of fuzziness and can effectively manage fuzzy conditions.

Definition 2. Grey numbers are numerical representations of uncertain information. White, black, and grey numbers serve as three distinct classifications for assessing information uncertainty. Let $\otimes G = [\underline{G}, \bar{G}] = \{x; : \underline{G} \leq x \leq \bar{G} \mid \underline{G}, \bar{G} \in \mathbb{R}\}$. Then $\otimes G$ which has two real parts $\underline{G}$ (the lower limit of $\otimes G$) and $\bar{G}$ (the upper limit of $\otimes G$) is defined as follows.

- i. If $\underline{G} \rightarrow -\infty$ and $\bar{G} \rightarrow \infty$, then $\otimes G$ is referred to as a black number, indicating the absence of meaningful information.

- ii. Else if $\underline{G} = \bar{G}$, then $\otimes G$ is known as a white number, signifying that the information is complete.
- iii. Otherwise, $\otimes G = [\underline{G}, \bar{G}]$ characterizes information that is neither fully known nor entirely unknown.
- iv. $\otimes G = (-\infty, \bar{G}]$ is called a lower-limit grey number.
- v. $\otimes G = [\underline{G}, \infty)$ is called an upper-limit grey number.

Definition 3. Operations on grey numbers are defined on set of intervals, not on real numbers. Let $+$, $-$, $\times$, and $\div$ indicate the operations of addition, subtraction, multiplication, and division, respectively. Let $\otimes x = [\underline{x}, \bar{x}]$, $\otimes y = [\underline{y}, \bar{y}]$ be two grey numbers. Then the arithmetic operations between $\otimes x, \otimes y$ are defined as follows:

- i. $\otimes x + \otimes y = [\underline{x} + \underline{y}, \bar{x} + \bar{y}]$
- ii. $\otimes x - \otimes y = [\underline{x} - \bar{y}, \bar{x} - \underline{y}]$
- iii. $\otimes x \times \otimes y = [\min\{\underline{x}\underline{y}, \bar{x}\bar{y}, \underline{x}\bar{y}, \bar{x}\underline{y}\}, \max\{\underline{x}\underline{y}, \bar{x}\bar{y}, \underline{x}\bar{y}, \bar{x}\underline{y}\}]$
- iv. $\otimes x \div \otimes y = \otimes x \times \left[\frac{1}{\bar{y}}, \frac{1}{\underline{y}}\right], 0 \notin \otimes y$.

Also

- v. $k \times \otimes x = [k\underline{x}, k\bar{x}]$.

3. Detailed review of some Literatures

The application of grey system theory in multi-attribute decision-making (MADM) has attracted attention because it enables the evaluation of alternatives when expert assessments and criterion values are uncertain or expressed linguistically. Existing studies have combined grey numbers with ideal-solution methods, ratio-based assessment, aggregation procedures, and approaches for determining decision-maker weights. The following review examines representative contributions that provide a basis for the comparative analysis presented in Section 7.

Tabor [47] investigated the use of grey-based TOPSIS for occupational risk management. Expert evaluations expressed as linguistic terms were represented by grey numbers and aggregated into a grey decision matrix. Following normalization and weighting, the method established ideal maximum and minimum solutions, measured each alternative's distance from these reference points, and derived closeness coefficients for ranking occupational risks. This application illustrates how grey TOPSIS can accommodate linguistic assessments in risk evaluation.

Zaheret *al.*, [46] proposed two grey-based decision-making methods, GMOOSRA and GOCRA. GMOOSRA extends simple ratio analysis by constructing, normalizing, and weighting a grey decision matrix, assessing benefit and cost criteria, and converting the resulting grey performance values into crisp scores through whitening. GOCRA instead derives preference ratings for non-beneficial and beneficial attributes, including their respective input and output criteria, before aggregating the ratings and applying a center-of-area conversion. Although both methods rely on grey evaluations, they differ in how they aggregate criterion performance and obtain final rankings.

Nyaoga *et al.*, [45] integrated grey system theory with TOPSIS to assess value chain performance. Their group decision-making framework begins with the selection of evaluation scales and the determination of criterion weights. Linguistic judgments are converted into grey numbers and organized into a weighted, normalized decision matrix. Positive and negative ideal solutions are then used to calculate separation measures and relative closeness values. The study demonstrates the use of grey TOPSIS for comparing performance across value chain alternatives.

Zavadskas *et al.*, [44] developed the WASPAS-G method for selecting construction contractors in accordance with stakeholder preferences and project requirements. The approach normalizes grey attribute values and combines weighted sum and weighted product optimality functions. An aggregation constant is determined by equating the total grey scores of the two components across alternatives, after which the combined function provides the basis for ranking. Unlike distance-based TOPSIS variants, WASPAS-G evaluates alternatives through aggregated performance functions.

Feng *et al.*, [48] introduced Grey Fuzzy TOPSIS (GPT), combining fuzzy measures and the Choquet integral to represent interactions among criteria. Linguistic evaluations are transformed into triangular fuzzy numbers and defuzzified before standardization and the determination of objective and subjective weights. Fuzzy measures support the calculation of grey-fuzzy correlation degrees, while distances to ideal solutions are used to obtain comprehensive proximity scores. This contribution is particularly relevant where criteria cannot be treated as fully independent.

Another research direction concerns the determination of decision-maker weights rather than only the ranking of alternatives. Yue [50] proposed an extended TOPSIS procedure in which linguistic assessments are converted into interval numbers, normalized, and weighted to form a weighted normalized interval decision matrix. The distances of individual decision-makers from positive and negative ideal solutions are used to calculate relative closeness values, which in turn determine their weights. In a related approach, Yue [52] incorporated left and right negative ideal solutions alongside the positive ideal solution and used the corresponding separation measures to derive decision-maker weights.

Eshlaghy and Farokhi [58] addressed decision-maker weighting through an iterative eigenvector approach based on pairwise comparisons and the principles of reciprocity and consistency. The decision matrix is repeatedly processed until convergence is approached, and the weight assigned to each decision-maker is calculated from the proportion of iterations associated with that decision-maker relative to the total number of iterations. This approach differs from TOPSIS-based weighting methods by relying on an iterative comparison procedure rather than ideal-solution distances.

Li *et al.*, [27] developed a grey-based supplier selection method involving multiple alternatives, attributes, and experts. Two seven-point linguistic scales were used to assess supplier performance and attribute importance. The interval grey evaluations were averaged, normalized, and weighted, and an ideal supplier was formed from the maximum grey values for each attribute. Grey possibility degrees were then used to compare suppliers with the ideal alternative, with smaller values indicating better performance. Jadidi *et al.*, [30] subsequently extended this line of work by introducing a negative ideal supplier based on minimum interval grey values and assessing alternatives through possibility degrees relative to both ideal references.

Overall, the reviewed studies can be grouped into grey TOPSIS and ideal-solution approaches [45, 47, 48], grey aggregation and ratio-based methods [44, 46], grey possibility-based supplier evaluation [27, 30], and procedures for estimating decision-maker weights [50, 52, 58]. Their principal differences concern the representation and aggregation of uncertainty, the treatment of criterion interactions, the construction of reference solutions, and the calculation of final ranking or weighting indices. These methodological distinctions provide the basis for the comparative analysis in Section 7.

4. Grey Possibility Degree and Grey Preference Degree

Ranking grey numbers over the real line is a critical aspect of decision-making. An alternative X is typically represented by a set of n grey numbers corresponding to n attributes. To compare X with the ideal alternative, their respective grey numbers are analyzed, and the ranking is derived from the comparison results. This section first defines the grey possibility degree, as referenced in [27], and

then introduces the grey preference degree for comparing two grey numbers over a finite scale on the real line.

Definition 4 ([27]). Given two grey numbers $\otimes G_1 = [\underline{G}_1, \bar{G}_1]$ and $\otimes G_2 = [\underline{G}_2, \bar{G}_2]$, the possibility degree of the event $\otimes G_1 \leq \otimes G_2$ is expressed as $P(\otimes G_1 \leq \otimes G_2) = \frac{\max(0, L^* - \max(0, \bar{G}_1 - \underline{G}_2))}{L^*}$

where $L^* = L(\otimes G_1) + L(\otimes G_2)$.

The above-mentioned grey possibility degree has the following properties.

1. If $\otimes G_1 = \otimes G_2$, then $P(\otimes G_1 \leq \otimes G_2) = 0.5$,
2. If $\underline{G}_2 > \bar{G}_1$, then $P(\otimes G_1 \leq \otimes G_2) = 1$,
3. If $\bar{G}_2 < \underline{G}_1$, then $P(\otimes G_1 \leq \otimes G_2) = 0$,
4. When there is some part in the intersection of $\otimes G_1$ and $\otimes G_2$, then $P(\otimes G_1 \leq \otimes G_2) < 0.5$ implies $\otimes G_2$ is smaller than $\otimes G_1$ and $P(\otimes G_1 \leq \otimes G_2) > 0.5$ implies $\otimes G_1$ is smaller than $\otimes G_2$.

This method of comparing interval grey numbers does not lead to an error free solution, and a serious drawback regarding this has been addressed in Mukherjee and Kar [21].

In this work, the authors introduce Grey Preference Degree between two IGNs as given in

Definition 2. We consider two grey numbers $\otimes G_1 = [\underline{G}_1, \bar{G}_1]$, $\otimes G_2 = [\underline{G}_2, \bar{G}_2]$ and two fixed real numbers a and b where $a \leq \min\{\underline{G}_1, \bar{G}_1, \underline{G}_2, \bar{G}_2\}$ and $b \geq \max\{\underline{G}_1, \bar{G}_1, \underline{G}_2, \bar{G}_2\}$. The grey preference degree of $\otimes G_1$ and $\otimes G_2$ means how much we should prefer $\otimes G_2$ than $\otimes G_1$, denoted by $P\{\otimes G_1, \otimes G_2\}$ and is defined by $P\{\otimes G_1, \otimes G_2\} = \frac{\sqrt{(b-\underline{G}_1)^2 + (b-\bar{G}_1)^2} - \sqrt{(b-\underline{G}_2)^2 + (b-\bar{G}_2)^2}}{\sqrt{2}(b-a)}$ over the scale $[a, b]$.

The value of $P\{\otimes G_1, \otimes G_2\}$ depends on the choice of the real numbers a and b . This should not be a problem in the process of solving a multi attribute decision making problem, because for a certain problem these values are fixed

Definition 3. Consider two sets of n interval grey numbers $(G_1) = \{\otimes G_1, \otimes G_2, \dots, \otimes G_n\}$ and $(G_2) = \{\otimes G'_1, \otimes G'_2, \dots, \otimes G'_n\}$. The grey preference degree of (G_1) and (G_2) is denoted by $P\{(G_1), (G_2)\}$ and is defined by $P\{(G_1), (G_2)\} = \frac{1}{n} [P(\otimes G_1, \otimes G'_1) + P(\otimes G_2, \otimes G'_2) + \dots + P(\otimes G_n, \otimes G'_n)]$.

Property (i). The range of values of $P\{\otimes G_1, \otimes G_2\}$ is $[-1, 1]$. When $P\{\otimes G_1, \otimes G_2\}$ is negative, we have to consider $P\{\otimes G_2, \otimes G_1\}$ where $P\{\otimes G_1, \otimes G_2\} = -P\{\otimes G_2, \otimes G_1\}$.

Property (ii). $P\{\otimes G_1, \otimes G_2\} = 1$, if $\otimes G_1$ coincides with a and $\otimes G_2$ coincides with b , i.e., if $\otimes G_1 = [a, a]$ and $\otimes G_2 = [b, b]$.

Property (iii). $P\{\otimes G_1, \otimes G_2\} = 0$ if and only if $\otimes G_1$ and $\otimes G_2$ are identical.

5. A novel method based on Grey Preference Degree

This method starts with the consideration of the alternative set $\{S_1, S_2, \dots, S_n\}$, the set of attributes $\{A_1, A_2, \dots, A_m\}$ and a group of p Decision Makers $\{D_1, D_2, \dots, D_p\}$ for suggesting their valuable decisions on the alternative rating and attribute rating.

Stage I: Calculation of weights of Decision Makers and Attributes. Weights of the decision makers

For the ranking of alternatives, the involvements of decision makers play an important role. Their views on different attributes influence the process of alternative selection. But as they may have been invited from different part of the society, their work experiences and views on the attributes may be different. So, it is quite unusual to consider the Decision Makers as equally important. For this reason, to make the process justified weights (crisp) are assigned to the decision makers.

The concerned agency/ department/ institution/ company must select a particular alternative from the list of alternatives they have or from the out of the list. Let that alternative be S_λ . The

compulsory condition is that they must have adequate information on S_λ over all the attributes and the decision vector on S_λ is said to be the ideal decision vector and let it be $V_D(S_\lambda)$. Then $V_D(S_\lambda) = \{S_{\lambda_1}^L, S_{\lambda_2}^L, \dots, S_{\lambda_m}^L\}$. The corresponding Grey Decision Vector is $G_{V_D}(S_\lambda)$. Then $G_{V_D}(S_\lambda) = \{G_{S_{\lambda_1}^L}, G_{S_{\lambda_2}^L}, \dots, G_{S_{\lambda_m}^L}\}$.

Now the appointed Decision Makers are asked to submit their decisions on S_λ over all the attributes. Let the decision vector of the k th Decision Maker on ideal alternative S_λ be $V_{D^K}(S_\lambda)$.

Then $V_{D^K}(S_\lambda) = \{S_{\lambda_1}^{L^K}, S_{\lambda_2}^{L^K}, \dots, S_{\lambda_m}^{L^K}\}$

The grey decision vector of $V_{D^K}(S_\lambda)$ is given by $G_{V_{D^K}}(S_\lambda) = \{G_{S_{\lambda_1}^{L^K}}, G_{S_{\lambda_2}^{L^K}}, \dots, G_{S_{\lambda_m}^{L^K}}\}$.

Now the weight of the k th Decision Maker is denoted by ω_k and is defined by

$$\omega_k = \frac{1 - \frac{|P\{G_{V_{D^K}}(S_\lambda), G_{V_D}(S_\lambda)\}|}{\sum_{k=1}^p |P\{G_{V_{D^K}}(S_\lambda), G_{V_D}(S_\lambda)\}|}}{\sum_{k=1}^p \left(1 - \frac{|P\{G_{V_{D^K}}(S_\lambda), G_{V_D}(S_\lambda)\}|}{\sum_{k=1}^p |P\{G_{V_{D^K}}(S_\lambda), G_{V_D}(S_\lambda)\}|} \right)},$$

By this process, the Decision Maker, whose decision on S_λ over all the attributes is closer to the decision of the organization, acquires better weight. These weights are normalized and so we must have $\sum_{k=1}^p \omega_k = 1$.

5.1 Weights of the Attributes

In this phase, the ordering of the alternatives is evaluated, based on a TOPSIS type approach. Like TOPSIS, positive and negative ideal alternatives are formed. But the comparison of each alternative with the PIA and NIA are executed by grey preference degree.

The attributes of the concerned procedure are very much important in the whole evaluation process. Decision makers are asked to submit their decisions on the Attribute Rating. Each attributes receives a linguistic term from the scale {Very High, High, Medium High, Medium, Medium Low, Very Low}. This is a 7 point scale and the corresponding interval grey numbers are shown in Table 1. Now from each Decision maker, a decision vector on attribute rating is formed and let it be $V_{D^K}(A) = \{A_1^{L^K}, A_2^{L^K}, \dots, A_m^{L^K}\}$.

Table 1
Scale of attribute weights

Scale	Grey Interval
VL	[0-0.1]
L	[0.1-0.3]
ML	[0.3-0.4]
M	[0.4-0.6]
MH	[0.6-0.7]
H	[0.7-0.9]
VH	[0.9-1]

The corresponding Grey Decision Vector will be $G_{V_{D^K}}(A) = \{G_{A_1^{L^K}}, G_{A_2^{L^K}}, \dots, G_{A_m^{L^K}}\}$. These p vectors together constitute a matrix, called Grey Attribute Decision Matrix (GADM). For computational simplicity, $G_{V_{D^K}}(A)$ is expressed as $\{(\underline{G}_{A_1}^k, \bar{G}_{A_1}^k), (\underline{G}_{A_2}^k, \bar{G}_{A_2}^k), \dots, (\underline{G}_{A_m}^k, \bar{G}_{A_m}^k)\}$ in terms of interval grey numbers.

Now the weights of the Decision Makers are considered and the ‘average aggregation principle’ is applied to obtain the grey weights of the attributes. If $\omega_{A_j}^G$ stands for the grey weight of the j th attribute, then $\omega_{A_j}^G$ is defined as $\omega_{A_j}^G = \left(\frac{1}{p} \sum_{k=1}^p \omega_k G_{A_j}^k, \frac{1}{p} \sum_{k=1}^p \omega_k \bar{G}_{A_j}^k \right)$, this is another interval grey number. Algorithms can be determined to obtain the weights by crisp numbers, but that will increase the computational error. Thus, throughout the methodology the number of transformations of grey numbers to crisp numbers should be intentionally decreased.

Stage II: Selection of Best Supplier and Classification of Suppliers. To select the best supplier the following steps are to follow:

Step 1. Formation of Grey Decision Matrix for Attributes (GDMA).

For this step Decision Makers are requested to submit their decisions on the rating of the alternatives by linguistic terms from the scale {Very Good, Good, Medium Good, Medium, Medium Poor, Poor, Very Poor}. This is another 7-point scale whose corresponding interval grey numbers are presented in Table 2.

Table 2

Scale of attribute ratings

Scale	Grey Interval
Very Poor	[0-1]
Poor	[1-3]
Medium Poor	[3, 4]
Fair	[4-6]
Medium Good	[6, 7]
Good	[7-9]
Very Good	[9, 10]

Let the form of Decision Vector of k th Decision Maker on the i th alternative S_i by

$$V_{D^K}(S_i) = \{S_{i1}^{L^K}, S_{i2}^{L^K}, \dots, S_{im}^{L^K}\}.$$

This is a m tuple vector, because of the m attributes. The corresponding Grey Decision Vector will be of the form

$$G_{V_{D^K}}(S_i) = \{G_{S_{i1}^{L^K}}, G_{S_{i2}^{L^K}}, \dots, G_{S_{im}^{L^K}}\}.$$

These km vectors, each of them being m tuple, together constitute a Non-Aggregated GDMA. For simplification, the grey decision vector can be re-expressed in terms of interval grey numbers as

$$\{(\underline{G}_{S_i}^k(A_1), \bar{G}_{S_i}^k(A_1)), (\underline{G}_{S_i}^k(A_2), \bar{G}_{S_i}^k(A_2)), \dots, (\underline{G}_{S_i}^k(A_m), \bar{G}_{S_i}^k(A_m))\}.$$

Now the decisions of the Decision Makers are aggregated again by their corresponding weights and by the average principle.

$$\text{Let } \underline{G}_{S_i}(A_j) = \frac{1}{p} \sum_{k=1}^p \omega_k \underline{G}_{S_i}^k(A_j), \bar{G}_{S_i}(A_j) = \frac{1}{p} \sum_{k=1}^p \omega_k \bar{G}_{S_i}^k(A_j) \text{ and}$$

$$G_{V_D}(S_i) = \{(\underline{G}_{S_i}(A_1), \bar{G}_{S_i}(A_1)), (\underline{G}_{S_i}(A_2), \bar{G}_{S_i}(A_2)), \dots, (\underline{G}_{S_i}(A_m), \bar{G}_{S_i}(A_m))\}.$$

These k grey decision vectors (each of them being m tuple) together form the Grey Decision Matrix for Alternatives (GDMA). So GDMA is a $m \times n$ matrix whose elements are interval grey numbers.

Step 2. Formation of Weighted GDMA and Normalized Weighted GDMA.

The elements of WGDMA are now weighted by the corresponding attributes’ weights. Let us denote it by

$$WG_{V_D}(S_i) =$$

$$\{(\omega_{A_1}^G \underline{G}_{S_i}(A_1), \omega_{A_1}^G \bar{G}_{S_i}(A_1)), (\omega_{A_2}^G \underline{G}_{S_i}(A_2), \omega_{A_2}^G \bar{G}_{S_i}(A_2)), \dots, (\omega_{A_m}^G \underline{G}_{S_i}(A_m), \omega_{A_m}^G \bar{G}_{S_i}(A_m))\} \quad \text{These } n$$

decision vectors (each of being m tuple) together constitute another $m \times n$ matrix, called Weighted GDMA (WGDMA).

From the WGDMA the maximum crisp member $GMAX = \max_{i,j} \{(\omega_{A_j}^G \underline{G}_{S_i}(A_j), \omega_{A_j}^G \bar{G}_{S_i}(A_j))\}$, is identified and the whole matrix is normalized by GMAX. Then the decision vector of the normalized WGDMA can be denoted by NWGDV(Si) and be expressed as

$$\left\{ \left(\frac{\omega_{A_1}^G \underline{G}_{S_i}(A_1)}{GMAX}, \frac{\omega_{A_1}^G \bar{G}_{S_i}(A_1)}{GMAX} \right), \left(\frac{\omega_{A_2}^G \underline{G}_{S_i}(A_2)}{GMAX}, \frac{\omega_{A_2}^G \bar{G}_{S_i}(A_2)}{GMAX} \right), \dots, \left(\frac{\omega_{A_m}^G \underline{G}_{S_i}(A_m)}{GMAX}, \frac{\omega_{A_m}^G \bar{G}_{S_i}(A_m)}{GMAX} \right) \right\}.$$

Again, for the simplicity of the computation for the next steps, let us denote NWGDV(Si) by $\otimes S_i$ $\left(\frac{\omega_{A_m}^G \underline{G}_{S_i}(A_m)}{GMAX}, \frac{\omega_{A_m}^G \bar{G}_{S_i}(A_m)}{GMAX} \right)$ by $(\otimes \underline{G}_{ij}, \otimes \bar{G}_{ij})$ and $(\otimes \underline{G}_{ij}, \otimes \bar{G}_{ij})$ by $\otimes G_{ij}$.

Then we have $\otimes S_i = \{\otimes G_{i1}, \otimes G_{i2}, \dots, \otimes G_{im}\}$.

Step 3. Calculation of PIA (Positive Ideal Alternative) and NIA (Negative Ideal Alternative).

Two imitation alternatives PIA and NIA are constructed in this step. The first one is assumed to be the best alternative in the current situation of decision making. In other words, PIA is the alternative of the ultimate choice. Similarly, NIA is the worst choice among the alternatives. Then the decision vectors of PIA and NIA are defined as

$$\begin{aligned} \otimes PIA &= \left\{ \left(\max_i \otimes \underline{G}_{i1}, \max_i \otimes \bar{G}_{i1} \right), \left(\max_i \otimes \underline{G}_{i2}, \max_i \otimes \bar{G}_{i2} \right), \dots, \left(\max_i \otimes \underline{G}_{im}, \max_i \otimes \bar{G}_{im} \right) \right\} \\ &= \{(\otimes G_{i1}^{MAX}, \otimes G_{i1}^{MAX}), (\otimes G_{i2}^{MAX}, \otimes G_{i2}^{MAX}), \dots, (\otimes G_{im}^{MAX}, \otimes G_{im}^{MAX})\} \text{ and} \end{aligned}$$

$$\begin{aligned} \otimes NIA &= \left\{ \left(\min_i \otimes \underline{G}_{i1}, \min_i \otimes \bar{G}_{i1} \right), \left(\min_i \otimes \underline{G}_{i2}, \min_i \otimes \bar{G}_{i2} \right), \dots, \left(\min_i \otimes \underline{G}_{im}, \min_i \otimes \bar{G}_{im} \right) \right\} \\ &= \{(\otimes G_{i1}^{MIN}, \otimes G_{i1}^{MIN}), (\otimes G_{i2}^{MIN}, \otimes G_{i2}^{MIN}), \dots, (\otimes G_{im}^{MIN}, \otimes G_{im}^{MIN})\}. \end{aligned}$$

Step 4. Relationship of each alternative from PIA and NIA by grey preference degree.

Each alternative, now, in this step, is compared with the PIA and NIA by Grey preference degree. Let the grey preference degree of Si and PIA be $P(\otimes S_i, \otimes PIA)$. It expresses the measure of preference of the PIA to Si.

$$\text{Then } P(\otimes S_i, \otimes PIA) = \frac{1}{m} \sum_{j=1}^m P(\otimes G_{ij}, \otimes G_{ij}^{MAX}).$$

Each alternative is now compared with the NIA. Let the grey preference degree of NIA and Si be $P(\otimes NIA, \otimes S_i)$. It expresses the measure of preference of Si than the NIA.

$$\text{Then } P(\otimes NIA, \otimes S_i) = \frac{1}{m} \sum_{j=1}^m P(\otimes G_{ij}^{MIN}, \otimes G_{ij}).$$

Step 5. assessment of the closeness co-efficient and the ranking

Closeness Co-efficient describes the relative closeness of each alternative with respect to the PIA and NIA. Let us now denote the Closeness Co-efficient of Si by CC(Si).

$$\text{Then } CC(Si) = \frac{P(\otimes NIA, \otimes S_i)}{P(\otimes NIA, \otimes S_i) + P(\otimes S_i, \otimes PIA)}.$$

The alternative whose CC is maximum, is the best choice and the ranking is done according to this principle.

5.2 Classification of the alternatives

The final phase classifies the alternatives in several groups. Its necessity lies in the fact that in the future dealing with the same alternatives should be influenced by their previous performances. This approach is novel. First, the number of such groups should be made fixed. The number should not exceed the number of linguistic terms in the alternative rating linguistic scale, provided in the methodology. As a particular case, six such groups are proposed: Totally Poor, Poor, Average, Good, Very Good and Excellent. For each of these six groups, one interval of real numbers is identified, that

corresponds to the closeness co-efficient of the groups. Let $[0, \delta_P], \delta_P, \delta_{MP}, \delta_{MP}, \delta_F, \delta_F, \delta_G, \delta_G, \delta_{VG}, [\delta_{VG}, 1]$ be the interval of values of closeness co-efficient of the said groups respectively.

A fictitious alternative S_P is constituted whose decision vector is assumed to be $V_D(S_P) = \{P, P, \dots, P\}$. the value of the closeness co-efficient of S_P will be considered as the value of δ_P . This is based on the hypothesis that a Poor alternative can mostly obtain this rating vector from the decision makers. So $\delta_P = CC(S_P)$. Similarly other fictitious alternatives S_{MP}, S_F, S_G and S_{VG} are constructed whose decision vectors are $V_D(S_{MP}) = \{MP, MP, \dots, MP\}$, $V_D(S_F) = \{F, F, \dots, F\}$, $V_D(S_G) = \{G, G, \dots, G\}$ and $V_D(S_{VG}) = \{VG, VG, \dots, VG\}$ respectively. So, we can summarize the classification as follows:

1. If $CC(S_i) \in [0, \delta_P]$ then S_i is Totally Poor.
2. If $CC(S_i) \in \delta_P, \delta_{MP}$ then S_i is Poor.
3. If $CC(S_i) \in \delta_{MP}, \delta_F$ then S_i is Average.
4. If $CC(S_i) \in \delta_F, \delta_G$ then S_i is Good.
5. If $CC(S_i) \in \delta_G, \delta_{VG}$ then S_i is Very Good.
6. If $CC(S_i) \in [\delta_{VG}, 1]$ then S_i is Excellent.

This completes the whole evaluation process. The working algorithm is briefly described as follows:

- i. Determine the decision vector.
- ii. Evaluate the weights of the decision makers.
- iii. Evaluate the weights of the attributes.
- iv. Form the Grey decision matrix for the alternatives.
- v. Form the weighted Grey decision matrix for the alternatives.
- vi. Form the normalized weighted Grey decision matrix for the alternatives.
- vii. Construct the ideal alternatives PIA and NIA.
- viii. Evaluate the closeness coefficient.
- ix. Classify the alternatives in the groups

6. Case Study: Suppliers Selection Problem

In this section the proposed approach is applied to a real case study in construction business. Ajoy Associates is one of the largest companies in Burdwan, West Bengal in construction business. They made different types of buildings (residential complex, Supermarket, Private hospital etc.). The one of the raw materials for building construction is brick. For this type of raw materials, the company needs suppliers. From the quotation of tender of four brick making factories, the company must select the best brick making factory (supplier). The four factories are Ratan Bhata (S_1), H.M.T industry (S_2), Power bricks (S_3), Biswas bricks (S_4). The supplied bricks must be of 3kg in weight and the dimensions should be 10 X 5 X 3 cubic cm. The company needs the best supplier that suits well with the four attributes: Product Quality (A_1), Service Quality (A_2), Delivery in time (A_3) and Price (A_4). A group of experts (D_1, D_2, D_3) has appointed by the company and they have been asked to submit their decision in terms of linguistic terms on the rating of the suppliers.

The choice of the linguistic terms of the decision makers about the suppliers with respect to the four attributes is displayed in Table 3.

Table 3
Linguistic Decision Matrix

Attributes	S_i	D_1	D_2	D_3
A_1	S_1	MG	G	MG
	S_2	F	F	G
	S_3	MG	G	G
	S_4	MP	F	F

Table 3
Continued

Attributes	S _i	D ₁	D ₂	D ₃
A ₂	S ₁	F	MG	F
	S ₂	MG	MG	G
	S ₃	G	G	G
	S ₄	MG	F	MG
A ₃	S ₁	F	MG	F
	S ₂	MG	F	G
	S ₃	G	MG	G
	S ₄	F	MG	G
A ₄	S ₁	MG	MG	MG
	S ₂	F	MP	MG
	S ₃	MG	F	G
	S ₄	F	F	MG

Weights of the decision makers

In this example, let us consider an ideal supplier, say S₅, which is used to determine the weights of the decision makers. The decision of the company (X) as well as the decision makers (D₁, D₂, D₃) on S₅ over all the attributes is given in Table 4.

Table 4
Linguistic terms of the decision of the decision makers and company on S5

	A ₁	A ₂	A ₃	A ₄
D ₁	G	VG	MP	MP
D ₂	VG	VG	P	MP
D ₃	G	G	P	F
X	G	MG	F	MP

Using the aforesaid algorithm, the weights of the decision makers are found as $w_{D_1} = 0.182$, $w_{D_2} = 0.289$, and $w_{D_3} = 0.529$.

Weights of the attributes

Here in this example, the weights of the attributes as given by the decision makers are displayed in Table 5.

Table 5
Attribute ratings by the decision makers

	D ₁	D ₂	D ₃
A ₁	H	M	MH
A ₂	M	H	H
A ₃	MH	MH	M
A ₄	H	M	VH

Now following proposed method, the weights of the attributes in terms of interval grey numbers are obtained as follows: weight of A₁ = [0.191, 0.246], weight of A₂ = [0.199, 0.266], weight of A₃ = [0.175, 0.221], and weight of A₄ = [0.227, 0.282].

Step 1. Formation of Grey Decision Matrix for Attributes (GDMA).

The grey numbers for the corresponding linguistic terms are substituted in Table 5 to form Table 6.

Step 2. Formation of Weighted GDMA and Normalized Weighted GDMA.

The corresponding GDMA and WGDMA are constructed as Table 7 and Table 8.

Table 6

Decisions in terms of interval grey numbers

Attributes	S_i	D_1	D_2	D_3
A_1	S_1	[6, 7]	[7, 9]	[6, 7]
	S_2	[4, 6]	[4, 6]	[7, 9]
	S_3	[6, 7]	[7, 9]	[7, 9]
	S_4	[3, 4]	[4, 6]	[4, 6]
A_2	S_1	[4, 6]	[6, 7]	[4, 6]
	S_2	[6, 7]	[6, 7]	[7, 9]
	S_3	[7, 9]	[7, 9]	[7, 9]
	S_4	[6, 7]	[4, 6]	[6, 7]
A_3	S_1	[4, 6]	[6, 7]	[4, 6]
	S_2	[6, 7]	[4, 6]	[7, 9]
	S_3	[7, 9]	[6, 7]	[7, 9]
	S_4	[4, 6]	[6, 7]	[7, 9]
A_4	S_1	[6, 7]	[6, 7]	[6, 7]
	S_2	[4, 6]	[3, 4]	[6, 7]
	S_3	[6, 7]	[4, 6]	[7, 9]
	S_4	[4, 6]	[4, 6]	[6, 7]

Table 7

GDMA

	A_1	A_2	A_3	A_4
S_1	[2.096, 2.526]	[1.526, 2.176]	[1.526, 2.176]	[2, 2.333]
S_2	[1.862, 2.529]	[2.176, 2.686]	[1.984, 2.590]	[1.590, 1.984]
S_3	[2.273, 2.879]	[2.333, 3]	[2.237, 2.807]	[1.984, 2.590]
S_4	[1.273, 1.879]	[1.807, 2.237]	[2.055, 2.625]	[1.465, 2.218]

Table 8

Weighted GDMA

	A_1	A_2	A_3	A_4
S_1	[0.400, 0.621]	[0.304, 0.579]	[0.267, 0.481]	[0.454, 0.658]
S_2	[0.356, 0.622]	[0.433, 0.714]	[0.347, 0.572]	[0.361, 0.559]
S_3	[0.434, 0.708]	[0.464, 0.798]	[0.391, 0.620]	[0.450, 0.730]
S_4	[0.243, 0.462]	[0.360, 0.595]	[0.360, 0.580]	[0.332, 0.625]

The normalized WGDMA is formed and displayed in Table 9 by taking $G^{\text{MAX}} = 0.798$.

Table 9

Normalized WGDMA

	A_1	A_2	A_3	A_4
S_1	[0.501, 0.778]	[0.381, 0.726]	[0.334, 0.603]	[0.568, 0.824]
S_2	[0.446, 0.779]	[0.543, 0.894]	[0.434, 0.717]	[0.452, 0.700]
S_3	[0.544, 0.887]	[0.581, 1.000]	[0.490, 0.777]	[0.731, 0.915]
S_4	[0.304, 0.579]	[0.451, 0.746]	[0.451, 0.727]	[0.416, 0.783]

Step 3. Calculation of PIA (Positive Ideal Alternative) and NIA (Negative Ideal Alternative).

In the present example, $\otimes PIA = \{[0.544, 0.887], [0.581, 1], [0.490, 0.777], [0.731, 0.915]\}$ and

$\otimes NIA = \{[0.304, 0.579], [0.381, 0.726], [0.334, 0.603], [0.416, 0.700]\}$. In the proposed example, $P(\otimes S_1, \otimes PIA) = 0.525$, $P(\otimes S_2, \otimes PIA) = 0.413$, $P(\otimes S_3, \otimes PIA) = 0.052$, $P(\otimes S_4, \otimes PIA) = 0.676$ and $P(\otimes NIA, \otimes S_1) = 0.323$, $P(\otimes NIA, \otimes S_2) = 0.155$, $P(\otimes NIA, \otimes S_3) = 0.962$, $P(\otimes NIA, \otimes S_4) = 0.354$.

Step 4. Relationship of each alternative from PIA and NIA by grey preference degree.

In the working example, $CC(S_1)=0.381$, $CC(S_2)=0.273$, $CC(S_3)=0.949$, $CC(S_4)=0.344$. So here the 3rd supplier, S_3 , is the best choice, and the order is of the form $S_3>S_1>S_4>S_2$.

Here in this example we get $\delta_P = 0$, $\delta_{MP} = 0$, $\delta_F = 0.025$, $\delta_G = 0.5729$, $\delta_{VG} = 1$.

So, in this particular case study, we have three intervals i.e. Average = (0, 0.025], Good = (0.025, 0.5729] and Good = (0.5729, 1]. Thus, we can say that S_1 , S_2 , S_4 are average suppliers but the supplier S_3 is good, or we can say excellent in this context.

7. Comparative analysis

Finally, a comparative analysis of methods discussed in section 3 is presented with identifying some advantages of using the proposed methodology based on the preference degree of Grey Numbers. Here we compare the proposed methodology with some existing approaches.

At first let us apply Tabor's approach [47] in the proposed case study. Since the weights of the DMs are not considered here, then the GDM of this method will be the same as shown in Table 10.

Table 10
GDMA

	A_1	A_2	A_3	A_4
S_1	(6.333, 7.667)	(4.667, 6.333)	(4.667, 6.333)	(6, 7)
S_2	(5, 7)	(6.333, 7.667)	(5.667, 7.333)	(4.333, 5.667)
S_3	(6.667, 8.333)	(7, 9)	(6.667, 8.333)	(5.667, 7.333)
S_4	(3.667, 5.333)	(5.333, 6.667)	(5.667, 7.333)	(4.667, 6.333)

The weights of the attributes are {(0.191, 0.246), (0.199, 0.266), (0.175, 0.221), (0.227, 0.282)}.

The weighted GDMA will be the same as shown in Table 11.

Table 11
WGDMA

	A_1	A_2	A_3	A_4
S_1	(1.210, 1.886)	(0.929, 1.684)	(0.817, 1.4)	(1.362, 1.974)
S_2	(0.955, 1.722)	(1.260, 2.039)	(0.992, 1.620)	(0.984, 1.598)
S_3	(1.273, 2.050)	(1.393, 2.394)	(1.167, 1.842)	(1.286, 2.068)
S_4	(0.700, 1.312)	(1.061, 1.773)	(0.992, 1.621)	(1.059, 1.786)

The maximum element in the WGDMA is 2.394. Then the normalized WGDMA will be the same as given in Table 12.

Table 12
NWGDMA

	A_1	A_2	A_3	A_4
S_1	(0.505, 0.788)	(0.388, 0.703)	(0.341, 0.585)	(0.569, 0.824)
S_2	(0.399, 0.719)	(0.526, 0.852)	(0.414, 0.677)	(0.411, 0.667)
S_3	(0.532, 0.856)	(0.582, 1)	(0.487, 0.769)	(0.537, 0.964)
S_4	(0.292, 0.548)	(0.443, 0.741)	(0.414, 0.677)	(0.442, 0.746)

Based on NWGDMA, the Ideal Max and Ideal Min alternatives are constructed as: $S_{MAX} = \{(0.532, 0.856), (0.582, 1), (0.487, 0.769), (0.569, 0.864)\}$ and $S_{MIN} = \{(0.292, 0.548), (0.388, 0.703), (0.341, 0.585), (0.411, 0.667)\}$.

Now the Closeness Co-efficients are calculated and are given as: $CC(S_1) = 0.389$, $CC(S_2) = 0.207$, $CC(S_3) = 0.788$, $CC(S_4) = 0.401$. So, the ranking according to this method is: $S_3>S_4>S_1>S_2$ which is different from the proposed result.

Next, we apply Zaheret *al.*, [46] approach in the case study described in section 6. At first, the weights of the attributes are calculated using SOWIA method which is based on standard derivation and sum weight method. The objective weights (O_j), subjective weight (S_j) and combined weights of the attributes are computed as: shown in Table 13.

Table 13
Values of O_j , S_j and w_j^{int}

	O_j	S_j	$w_j^{int} = 0.5 O_j + 0.5 S_j$
A_1	0.220	0.2	0.210
A_2	0.242	0.3	0.271
A_3	0.204	0.1	0.152
A_4	0.268	0.4	0.334

Now applying GOCRA method, overall grey preference $\otimes P_i$ values are calculated, the degreying crisp output y_i are calculated. All the values along with corresponding ranks are provided in Table 14.

Table 14
Ranking for GOCRA and GMOOSRA

Alt.	GOCRA		GMOOSRA	
	$\otimes P_i$	Rank	y_i	rank
S_1	0.688	3	0.288	4
S_2	0.614	4	0.301	3
S_3	0.714	1	0.418	1
S_4	0.701	2	0.343	2

So the ranking for GOCRA is $S_3 > S_4 > S_1 > S_2$ and for GMOOSRA is $S_3 > S_4 > S_2 > S_1$

Now the method used in Nyaoga *et al.*, [45] approach is applied in the case study described in section 6. After degreying the GDM will be as shown in Table 15.

Table 15
GDM

	A_1	A_2	A_3	A_4
S_1	7	5.5	5.5	6.5
S_2	6	7	6.5	5.5
S_3	7.5	8	7.5	6.5
S_4	4.5	6	6.5	5.5

The weights of the attributes after defraying will be:

$$w_{A_1} = \frac{0.191+0.246}{2} = 0.218, w_{A_2} = \frac{0.199+0.266}{2} = 0.232$$

$$w_{A_3} = \frac{0.175+0.221}{2} = 0.198, w_{A_4} = \frac{0.227+0.282}{2} = 0.254.$$

Then the weighted GDMA will be as shown in Table 16.

Table 16
Weighted GDM

	A_1	A_2	A_3	A_4
S_1	1.529	1.276	1.089	1.651
S_2	1.311	1.624	1.287	1.397
S_3	1.639	1.856	1.485	1.651
S_4	0.983	1.392	1.287	1.397

The maximum element in the WGDMA is 1.856. So the normalized WGDMA will be as shown in Table 17.

Table 17
Normalized WGDMA

	A ₁	A ₂	A ₃	A ₄
S ₁	0.824	0.687	0.587	0.890
S ₂	0.706	0.875	0.693	0.753
S ₃	0.883	1	0.800	0.890
S ₄	0.530	0.750	0.693	0.753

According to the methodology, the S_{MAX} and S_{MIN} will be: S_{MAX} = (0.883, 1, 0.8, 0.890) and S_{MIN} = (0.530, 0.687, 0.587, 0.753).

Thus, the separation values will be as shown in Table 18.

Table 18
Separation values

	+ Values (S _i ⁺)	-Values (S _i ⁻)
S ₁	0.383	0.324
S ₂	0.278	0.278
S ₃	0	0.535
S ₄	0.467	0.123

Now the relative closeness values are CC(S₁)= 0.458, CC(S₂)= 0.5, CC(S₃)= 1, CC(S₄)= 0.208. Thus, the ranking will be S₃ > S₂ > S₁ > S₄, which is not identical as in the case study.

Next, we apply Liet *et al.*, [27] approach in our proposed case study. As per the methodology, the GDM, WNGDM and NWGDM are identical with Table 12, Table 13 and Table 15.

Now S^{Max} = {(0.532, 0.856), (0.582, 1), (0.487, 0.769), (0.569, 0.864)}.

In the next step, Grey Possibility degree of each alternative and S^{Max} is calculated.

$$P\{S_1 \leq S^{Max}\} = \frac{1}{4}(0.796 + 0.711 + 0.806 + 0.822) = 0.784,$$

$$P\{S_2 \leq S^{Max}\} = \frac{1}{4}(0.798 + 0.701 + 0.791 + 0.821) = 0.778,$$

$$P\{S_3 \leq S^{Max}\} = \frac{1}{4}(0.766 + 0.617 + 0.775 + 0.772) = 0.732, \text{ and}$$

$$P\{S_4 \leq S^{Max}\} = \frac{1}{4}(0.816 + 0.727 + 0.791 + 0.788) = 0.780.$$

So the ranking is S₃ > S₂ > S₄ > S₁ which is different from the result obtained in the proposed case study.

Finally, the case study is compared with Jadidi *et al.*, [30] approach. As per the methodology, the GDM, WNGDM and NWGDM are identical with Table 11, Table 12 and Table 15.

Now S^{Max} = {(0.532, 0.856), (0.582, 1), (0.487, 0.769), (0.569, 0.864)}, and S^{Min} = {(0.292, 0.548), (0.388, 0.703), (0.487, 0.769), (0.569, 0.864)}. Here the Tao values are respectively 0.318, 0.422, 0.215 and 0.412. So, the ranking is S₃ > S₁ > S₄ > S₂.

Now we apply [50] in the proposed case study in Section 6. Considering the separation measures, the weights of the DMs will be as given in Table 19.

Table 19
Evaluation of the weights of the DMs by the method described in [50]

	A ₁	A ₂	A ₃	A ₄	Separation Values	Weights
D ₁	G	VG	MP	MP	6	0.3
D ₂	VG	VG	P	MP	6	0.3
D ₃	G	G	P	F	8	0.4

The weights are not same. So, after applying these weights in the GDM, the ranking of the suppliers is: $S_3 > S_2 > S_4 > S_1$ which is different from the proposed case study. Now, if we do not apply the weights of the decision makers in the case study provided in section 6, i.e., if we skip phase 1 in the methodology of this work, the result is the same, i.e., the ranking is $S_3 > S_2 > S_4 > S_1$.

Next, the methodology in [52] is applied in the proposed case study in Section 6. Table 20 shows the procedure of the evaluation of the weights of the DMs.

Table 20

Evaluation of the weights of the DMs by the method described in [52]

	A ₁	A ₂	A ₃	A ₄	Separation Values			Weights
					From PIS	From L-NIS	From R-NIS	
D ₁	G	VG	MP	MP	6	8	8	0.343
D ₂	VG	VG	P	MP	6	8	8	0.343
D ₃	G	G	P	F	8	6	10	0.314

In this case, the weights of the D₂ and D₃ are same. Again, if we apply these weights to the rest of the methodology, the ranking is $S_3 > S_2 > S_4 > S_1$, which is completely different from the ranking obtained in the case study.

Despite their contributions to grey-based multi-criteria decision-making, the reviewed approaches have several methodological limitations. In Tabor's approach [47], grey numbers in the final decision matrix are compared exclusively using distance measures, which may not fully reflect the uncertainty represented by these numbers. Zaher *et al.*, [46] did not provide a clearly defined procedure for determining the parameter α , which may affect the consistency and reproducibility of the results. In the method proposed by Nyaoga *et al.*, [45], the whitening of grey numbers is performed at an early stage, potentially reducing the ability of the model to preserve uncertainty throughout the decision-making process. The limitations of Li *et al.*'s approach [27] have already been discussed in Section 4. Similarly, Jadidi *et al.*, [30] adopted the possibility measure introduced by Li *et al.*, [27], thereby retaining the limitations associated with that measure.

Additional limitations can be identified in methods developed for determining the weights of decision-makers. In Yue's approach [50], the average decision of all decision-makers is assumed to represent the best decision, although this assumption may not always be appropriate. Furthermore, the weights are calculated using separation measures between individual decisions and the average decision, which may produce the same aggregated result as an approach that does not explicitly consider decision-maker weights. In the method proposed by Eshlaghy and Farokhi [58], the iterative procedure is not sufficiently defined, particularly regarding the number of iterations, the convergence criterion, and the selection of the convergence vector. Moreover, the convergence vector does not explicitly account for uncertainty. In another approach proposed by Yue [52], decision-maker weights are determined using separation measures from the average, maximum, and minimum decisions. However, this procedure may assign lower weights to highly experienced or competent decision-makers whose evaluations differ substantially from those of the group. Consequently, the resulting weights may not adequately represent the actual expertise and decision-making capabilities of individual decision-makers.

These limitations highlight the importance of developing decision-making methods that preserve uncertainty throughout the evaluation process, provide clearly defined computational procedures, and determine decision-maker weights without relying exclusively on their agreement with group-level evaluations.

The comparison of different approaches and their ranking of alternatives is as follows, Table 21.

Table 21

Comparison of different approaches and their ranking of alternatives

Methods	Way of Representing Linguistic Terms	Whether DMS' weights are considered	Whether compared to the NIS	Whether classified	Ranking
Li et al. [27]	IGN	No	No	No	$S_3 > S_2 > S_4 > S_1$
Jadidi et al. [30]	IGN	No	Yes	No	$S_3 > S_1 > S_4 > S_2$
Nyaoga et al. [45]	IGN	No	Yes	No	$S_3 > S_2 > S_1 > S_4$
Zaher et al. [46]	IGN	No	Not Applicable	No	$S_3 > S_4 > S_1 > S_2$ $S_3 > S_4 > S_2 > S_1$
Tabor [47]	IGN	No	Yes	No	$S_3 > S_4 > S_1 > S_2$
Yue [50]	Interval Numbers	Yes	Yes	No	$S_3 > S_2 > S_4 > S_1$
Yue [52]	Crisp numbers	Yes	Yes	No	$S_3 > S_2 > S_4 > S_1$
Proposed approach	IGN	Yes	Yes	Yes	$S_3 > S_1 > S_4 > S_2$

The comparative analysis highlights how the limitations of existing methodologies may influence the final decision-making results. Based on the findings, the proposed method offers several methodological advantages over the reviewed approaches. First, the weights of decision-makers are determined by calculating the total deviation between individual decisions and predefined ideal decisions, providing a systematic basis for evaluating their relative importance. Second, the algorithm is designed to reduce potential bias in the decision-making process. Third, the methodology incorporates a classification procedure that allows alternatives to be grouped according to their performance. This feature is particularly useful in practical applications, such as supplier selection, where companies can identify and shortlist suitable suppliers for future consideration. Finally, the proposed approach introduces an improved procedure for comparing Interval Grey Numbers (IGNs), addressing some of the limitations associated with existing comparison measures.

Overall, these advantages indicate that the proposed method provides a more comprehensive framework for handling uncertainty, determining decision-maker weights, and evaluating alternatives. By integrating weighting, ranking, and classification procedures, the methodology extends beyond the conventional ranking-oriented approaches discussed in the literature. The comparative analysis therefore suggests that the proposed method has the potential to improve the consistency and practical applicability of group decision-making under uncertainty.

8. Conclusion

This study introduced a three-phase group decision-making method for evaluating and classifying alternatives under uncertainty. The proposed approach considers evaluations provided by decision-makers in the form of linguistic variables, which are represented using Interval Grey Numbers (IGNs). In the first phase, the weights of decision-makers are determined to reflect their relative importance in the decision-making process. The second phase focuses on comparing IGNs and addressing the limitations of existing comparison measures. For this purpose, a Grey Preference Degree is introduced, and its effectiveness is examined through a detailed comparative analysis. In the third phase, the alternatives are classified according to their evaluation results. The applicability of the proposed methodology is demonstrated through a real-world supplier selection problem. In addition, the method can be applied to other decision-making problems involving uncertainty, including the identification of disadvantaged households, teacher performance evaluation, and student merit assessment. The classification procedure may also be useful in medical applications, where patients suffering from a particular disease can be classified into different categories based on their responses to relevant evaluation attributes.

Despite these contributions, the proposed methodology has certain limitations that should be addressed in future research. First, the evaluations provided by decision-makers are based on the historical performance of alternatives. Consequently, difficulties may arise when a new alternative is introduced without sufficient historical information or previous evaluations. Second, the proposed method does not explicitly consider interdependencies among evaluation attributes, which may influence the final results in complex decision-making environments. Future research should therefore focus on extending the methodology to accommodate alternatives with limited or unavailable historical data and incorporating relationships among attributes into the evaluation framework. Addressing these limitations could further improve the flexibility, reliability, and applicability of the proposed approach to a wider range of decision-making problems under uncertainty.

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Conflicts of Interest

The authors declare no conflicts of interest.

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