Journal of Contemporary Decision Science

Journal homepage: www.cds-journal.org/
ISSN: 3104-3380

Ranking Higher Education Institutions Using Entropy–VIKOR with Generalized Pentagonal Intuitionistic Fuzzy Numbers

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ARTICLE INFO

Article history:
Received 21 October 2025
Received in revised form 3 December 2025
Accepted 30 December 2025
Available online 4 January 2026

Keywords:
Higher educational institution ranking; Generalized pentagonal intuitionistic fuzzy numbers; Fuzzy sets; MCDM; Decision making; Entropy; VIKOR

ABSTRACT

Ranking the higher educational institutes is a very challenging task. In ranking higher educational institutions, many criteria and sub-criteria must be considered. These criteria may be beneficial or non-beneficial, which also increases the problem's complexity. Here, our aim is to develop an advanced framework to determine the rankings of higher educational institutions, incorporating uncertainty in the environment and the dataset. Here, we will take the help of Generalized Pentagonal Intuitionistic Fuzzy Numbers (GPIFNs), which are an extension of Pentagonal Fuzzy Numbers (PFNs), to capture the uncertainty of the model. For this purpose, we will include two popular MCDM methodologies: the Entropy weighted method for evaluating the criteria weights and the VIKOR method for ranking alternatives across different higher educational institutions. In this model, the opinions of different decision experts will be taken in linguistic terms as a dataset and will further be converted to GPIFNs. Lastly, sensitivity analysis and comparative analysis will be performed to assess the stability and robustness of the results obtained with this model. This model will be very effective for making policies for the advancement of higher educational institutions, and it will provide a clear view of the strengths and weaknesses of an institution to the administrator so that they can take necessary strategies to improve the quality of education.

1. Introduction

Education plays a crucial role in the overall development of a society. A progressive country always pays attention to educating its people. Education is not only important for gaining knowledge and developing ethics, but also helps to change the economic status of people. After finishing basic education, students get the opportunity to select their stream for higher study, which helps them in their future career. There are many higher educational institutions, some of which are government sponsored and some are private. There are many factors on which the reputation and popularity of an educational institution depend. During taking admission, students want to get the best opportunity for their study and so they want to choose an institution among those institutions which

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can provide the best educational facilities. Thus, ranking of higher educational institutions is very much needed.

1.1 Upgradation and competition in higher education

Upgradation and competition in higher education are essential to improve the academic standards and performance of institutions. As technology advances rapidly day by day, the demand for skilled students is increasing. Thus, higher educational institutions are focused on modernizing their techniques of teaching, adaptation of digital learning tools, advancement of facilities for research and invention, improvement in infrastructure, etc., so that they can achieve the global standards. As student try to get admission in a reputed institution having all types of facilities, there becomes a healthy competition among the institutions which pushes them to improve their facilities, quality of teaching, learning and skill development programs. Through upgradation, an institution becomes able to provide educational facilities which will be relevant to global needs and competition helps to maintain the excellence standard and reputation of the institution.

1.2 Impact of internationalization in university rankings

To improve the ranking of an institution globally, internationalization plays a crucial role. Students coming from different countries are trying to take admission in a reputed institution which possesses a good ranking in a global aspect. Internationalization of an educational institution shows how an institution attracts students from different countries and how the institution collaborates with foreign universities for research and development related work. Through internationalization, an institution can provide lecture classes and organize international conferences where faculty come from different countries to share their knowledge. The academic quality and global prestige of educational institutions are enhanced when they participate in international programs and work together with foreign universities on many projects. A research publication which is coming out from a research collaboration with a foreign university has a better possibility of getting more citations and global visibility.

1.3 Motivation of this study

After completing the basic education, student have the opportunity to choose their future path. Not all of the students want to pursue higher study; some students want to join short-term skill developing programs and some of them want to join their family business. But there are many students who have a great urge to pursue higher studies in a reputed educational institution. Everyone wants to choose the best option for their higher study according to their academic standard. For this purpose, it is necessary to give ranking to the higher educational institutions, but there are many factors, such as academic excellence, infrastructure, expenses, etc., on which this process is dependent. Therefore, there is a need to formulate a decision-making model to rank the higher educational institutions by including many conflicting criteria, some of which are beneficial and some are of a non-beneficial nature. In this paper, our aim is to construct a model where Generalized Pentagonal Intuitionistic Fuzzy Numbers (GPIFNs) are used to incorporate uncertainty and an Entropy-VIKOR based MCDM technique is applied to evaluate the ranking of higher educational institutions.

1.4 Research outline

Nowadays, to match the demand for students to get admission in a higher educational institution, the number of higher educational institutions has also increased. All institutes are trying to provide the best facilities for study to students. So, it is a challenging task for students to select one institute among them for further study. In this paper, our aim is to develop a framework to rank higher educational institutions. There are many factors on which the ranking of an institution depends, and we will select some of these important factors as criteria. Further, we will apply two MCDM methods, such as the entropy for evaluating the criteria weight and the VIKOR method, to perform the ranking of alternatives. In this framework, to incorporate uncertainty in the dataset, the generalized pentagonal intuitionistic fuzzy numbers (GPIFNs) will be included. Lastly, to assess the robustness and stability of the results, we will perform sensitivity and comparative analyses.

1.5 Structure of this paper

This subsection focuses on describing the structure of this paper. Section 1 represents the introduction, the impact of internationalization on university rankings, the motivation of this study, and the research outline of this work. In Section 2, a brief literature survey on the ranking of higher educational institutions, generalized pentagonal intuitionistic fuzzy numbers (GPFINs) and MCDM methods are given. In Section 3, the preliminaries of fuzzy sets and GPFINs are discussed. Two MCDM methods, namely the entropy-weighted method and the VIKOR method, are described in Section 4. In Section 5, a brief discussion about the considered criteria and alternatives is performed. The model construction and data collection from 3 decision experts are given in Section 6. Numerical illustration is discussed in Section 7. Sensitivity analysis and Comparative analysis are performed in Section 8. Finally, the conclusions are drawn and the future research scopes are discussed in Section 9.

2. Literature survey of this study

In this section, we will provide a brief literature review for this study. At first, literature on the application of Ranking of Higher Educational Institutions, then an introduction of generalized pentagonal intuitionistic fuzzy numbers (GPFINs) and finally the methodologies of MCDM will be discussed.

2.1 Literature on the role of ranking of higher educational institutions

Choosing a particular educational institution for further higher study is a very difficult task. There are many facts which can be taken into consideration for ranking higher educational institutions. In 2008, Charon *et al.,* [1] introduced a new tool in their work to evaluate higher education in Europe. Harvey [2] performed a critical review of the ranking of higher education institutions. Hou *et al.,* [3] in their study investigated the contribution of indicators in the ranking of universities in world university ranking systems. Chowdhury *et al.,* [4] in their study performed a comparative study of Global Ranking frameworks and indicators related to higher educational institutions. Kumar *et al.,* [5] in their work focused on finding the objectivity in performance ranking of higher education institutions using the DDEA method. Thomas [6] in his study tried to find the impact of a higher ranking of an educational institution on student satisfaction. Johnes [7] tried to focus on the actual meaning of university ranking in his work. The limitations, legitimacy and value conflict of the Berlin Principles on Ranking Higher Education Institutions are discussed by Barron [8].

2.2 Literature on the generalized pentagonal intuitionistic fuzzy numbers

Fuzzy numbers are applied in many research works to include uncertainty in decision-making problems. Zadeh [9] introduced Fuzzy sets in 1965. Pentagonal Fuzzy Number (PFN) is an extension of fuzzy numbers. A ranking method was proposed by Srinivasan *et al.*, [10] to solve the transformation problem, which included pentagonal fuzzy numbers. Chakraborty *et al.,* [11] in their work used a pentagonal fuzzy number and implemented it in game problems. Razzaq *et al.,* [12] in their study applied Pentagonal Intuitionistic Fuzzy Numbers. Umamageswari [13] applied generalized interval valued pentagonal fuzzy numbers (GIVPFNs) to tackle a fuzzy assignment problem. The GPFIN is an extension of the pentagonal fuzzy number (PFN). Raut *et al.,* [14] applied Intuitionistic Interval-Valued Pentagonal Fuzzy Numbers in their work.

2.3 Literature of the MCDM methodologies

Multi-Criteria Decision Making (MCDM) is an effective tool for dealing with complicated decision-making problems. In this paper, we have considered two popular MCDM methods, such as the entropy-weighted method and the VIKOR method. Here, we provide a brief literature review of these two MCDM methods. Yang *et al.,* [15] used an entropy-based approach for designing a data collection network. Chuansheng *et al.,* [16] applied the entropy method for safety evaluation of the smart grid. The entropy method is applied by Gorgij *et al.,* [17] for assessing the quality of irrigation water. Rahimi *et al.,* [18] used the entropy method for supplier selection.

Shemshadi *et al.,* [19] applied a fuzzy-based VIKOR method for supplier selection. Jahan [20] used the VIKOR method for material selection problems. Gao *et al.,* [21] included the VIKOR method to rank concrete bridge repair projects. An extension of the VIKOR method on hesitant fuzzy sets was performed by Zhang *et al.,* [22] for a decision-making problem. Perdana *et al.,* [23] used the VIKOR method to analyse college rankings.

3. Preliminaries of mathematical tools

In this section, we will discuss the mathematical tools in detail. In this paper, we will use generalized pentagonal intuitionistic fuzzy numbers (GPIFNs) to include uncertainty in the dataset. Pentagonal intuitionistic fuzzy number (PIFN) [12] is an extension of fuzzy number and generalized pentagonal intuitionistic fuzzy number (GPIFN) is a further extension of pentagonal intuitionistic fuzzy number (PIFN).

3.1 Fuzzy set and its extension

Fuzzy Set theory was introduced in 1965 by Lotfi A. Zadeh. It is highly effective for addressing real-world, complex decision-making problems that incorporate human judgment.

Definition 1. Let $\mathfrak{X}$ be a universal set. A fuzzy set $\tilde{\mathcal{F}}$ defined on $\mathfrak{X}$ can be represented as

$$\tilde{\mathcal{F}} = \{(\mathfrak{x}, \mu_{\tilde{\mathcal{F}}}(\mathfrak{x})) : \mathfrak{x} \in \mathfrak{X}\} \tag{1}$$

where $\mu_{\tilde{\mathcal{F}}}(\mathfrak{x})$ shows the membership value of $\mathfrak{x}$ in $\tilde{\mathcal{F}}$ and $\mu_{\tilde{\mathcal{F}}}(\mathfrak{x}) \in [0,1]$ for all $\mathfrak{x} \in \mathfrak{X}$.

Definition 2. A fuzzy set $\tilde{\mathcal{F}}$ is called a fuzzy number if it is defined on the set of real numbers $(\mathbb{R})$ and satisfies the following four conditions

i. $\tilde{\mathcal{F}}$ must have a bounded support.
ii. $\tilde{\mathcal{F}}$ is a convex fuzzy set.
iii. $\tilde{\mathcal{F}}$ is a normalized fuzzy set. Thus, $\exists \mathfrak{x} \in \mathbb{R}$ for which $\mu_{\tilde{\mathcal{F}}}(\mathfrak{x}) = 1$ holds.
iv. The membership function $\mu_{\tilde{\mathcal{F}}}(\mathfrak{x})$ is piecewise continuous.

An intuitionistic fuzzy set is an extension of a Fuzzy set, where every element has two membership functions, namely the membership function or degree of belongingness and the non-membership function or degree of non-belonginess.

Definition 3. An Intuitionistic fuzzy set $\widetilde{\mathcal{B}}$ defined on the universal set $\mathfrak{X}$ can be represented as

$$\widetilde{\mathcal{B}} = \langle \mathfrak{x}, \mu_{\widetilde{\mathcal{B}}}(\mathfrak{x}), \vartheta_{\widetilde{\mathcal{B}}}(\mathfrak{x}) \rangle \tag{2}$$

where the membership function $\mu_{\widetilde{\mathcal{B}}}(\mathfrak{x}) : \mathfrak{X} \to [0,1]$ and non-membership function $\vartheta_{\widetilde{\mathcal{B}}}(\mathfrak{x}) : \mathfrak{X} \to [0,1]$ satisfies the condition $0 \leq \mu_{\widetilde{\mathcal{B}}}(\mathfrak{x}) + \vartheta_{\widetilde{\mathcal{B}}}(\mathfrak{x}) \leq 1$, for all $\mathfrak{x} \in \mathfrak{X}$.

3.2 Pentagonal fuzzy number and its extension

There are varieties of fuzzy numbers which have been developed to capture the uncertainty more accurately. Depending on the nature of the graphical representation of the membership function,

fuzzy numbers are categorized into different types of fuzzy numbers, such as triangular fuzzy numbers (TFNs), trapezoidal fuzzy numbers (TrFNs), pentagonal fuzzy numbers (PFNs) [24], etc.
Definition 4. A fuzzy number $\widetilde{\wp} = (\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5)$ is said to be a Pentagonal Fuzzy Number (PFN) [24] if its membership function can be represented as

$$\mu_{\widetilde{\wp}}(\mathfrak{x}) = \begin{cases} 0 & ;\ for\ \mathfrak{x} < \mathcal{p}_1, \mathfrak{x} \geq \mathcal{p}_5 \\ \frac{(\mathfrak{x}-\mathcal{p}_1)}{(\mathcal{p}_2-\mathcal{p}_1)}; & for\ \mathcal{p}_1 \leq \mathfrak{x} \leq \mathcal{p}_2 \\ \frac{(\mathfrak{x}-\mathcal{p}_2)}{(\mathcal{p}_3-\mathcal{p}_2)}; & for\ \mathcal{p}_2 \leq \mathfrak{x} \leq \mathcal{p}_3 \\ 1; & for\ \mathfrak{x} = \mathcal{p}_3 \\ \frac{(\mathcal{p}_4-\mathfrak{x})}{(\mathcal{p}_4-\mathcal{p}_3)}; & for\ \mathcal{p}_3 \leq \mathfrak{x} \leq \mathcal{p}_4 \\ \frac{(\mathcal{p}_5-\mathfrak{x})}{(\mathcal{p}_5-\mathcal{p}_4)}; & for\ \mathcal{p}_4 \leq \mathfrak{x} \leq \mathcal{p}_5 \end{cases} \tag{3}$$

where $\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5 \in \mathbb{R}$.

Definition 5. Let $\tilde{\mathcal{P}} = \langle(\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5)\rangle$ be a Pentagonal Intuitionistic Fuzzy Number (PIFN) with the membership function $\mathfrak{T}_{\tilde{\mathcal{P}}}(\mathfrak{x})$ and non-membership function $\mathfrak{F}_{\tilde{\mathcal{P}}}(\mathfrak{x})$ which satisfies the condition that $\mathfrak{T}_{\tilde{\mathcal{P}}}(\mathfrak{x}) + \mathfrak{F}_{\tilde{\mathcal{P}}}(\mathfrak{x}) \leq 1$ and they are described as follows

$$\mathfrak{T}_{\tilde{\mathcal{P}}}(\mathfrak{x}) = \begin{cases} 0 & ;\ for\ \mathfrak{x} < \mathcal{p}_1, \mathfrak{x} \geq \mathcal{p}_5 \\ \frac{(\mathfrak{x}-\mathcal{p}_1)}{(\mathcal{p}_2-\mathcal{p}_1)}; & for\ \mathcal{p}_1 \leq \mathfrak{x} \leq \mathcal{p}_2 \\ \frac{(\mathfrak{x}-\mathcal{p}_2)}{(\mathcal{p}_3-\mathcal{p}_2)}; & for\ \mathcal{p}_2 \leq \mathfrak{x} \leq \mathcal{p}_3 \\ 1; & for\ \mathfrak{x} = \mathcal{p}_3 \\ \frac{(\mathcal{p}_4-\mathfrak{x})}{(\mathcal{p}_4-\mathcal{p}_3)}; & for\ \mathcal{p}_3 \leq \mathfrak{x} \leq \mathcal{p}_4 \\ \frac{(\mathcal{p}_5-\mathfrak{x})}{(\mathcal{p}_5-\mathcal{p}_4)}; & for\ \mathcal{p}_4 \leq \mathfrak{x} \leq \mathcal{p}_5 \end{cases} \tag{4}$$

and

$$\mathfrak{F}_{\tilde{\mathcal{P}}}(\mathfrak{x}) = \begin{cases} 1 & ;\ for\ \mathfrak{x} < \mathcal{p}_1, \mathfrak{x} \geq \mathcal{p}_5 \\ \frac{(\mathcal{p}_2-\mathfrak{x})}{(\mathcal{p}_2-\mathcal{p}_1)}; & for\ \mathcal{p}_1 \leq \mathfrak{x} \leq \mathcal{p}_2 \\ \frac{(\mathcal{p}_3-\mathfrak{x})}{(\mathcal{p}_3-\mathcal{p}_2)}; & for\ \mathcal{p}_2 \leq \mathfrak{x} \leq \mathcal{p}_3 \\ 0; & for\ \mathfrak{x} = \mathcal{p}_3 \\ \frac{(\mathfrak{x}-\mathcal{p}_3)}{(\mathcal{p}_4-\mathcal{p}_3)}; & for\ \mathcal{p}_3 \leq \mathfrak{x} \leq \mathcal{p}_4 \\ \frac{(\mathfrak{x}-\mathcal{p}_4)}{(\mathcal{p}_5-\mathcal{p}_4)}; & for\ \mathcal{p}_4 \leq \mathfrak{x} \leq \mathcal{p}_5 \end{cases} \tag{5}$$

where $\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5 \in \mathbb{R}$.

Definition 6. Let $\tilde{\mathcal{P}} = \langle(\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5), \sigma_{\tilde{\mathcal{P}}}, \tau_{\tilde{\mathcal{P}}}\rangle$ be a generalized pentagonal intuitionistic fuzzy number (GPIFN), where $\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5$ are five parameters used to define a pentagonal fuzzy number, membership function $\mathfrak{T}_{\tilde{\mathcal{P}}}(\mathfrak{x})$ and non-membership function $\mathfrak{F}_{\tilde{\mathcal{P}}}(\mathfrak{x})$ are described as follows

$$\mathfrak{T}_{\tilde{\mathcal{P}}}(\mathfrak{x}) = \begin{cases} 0 & ;\ for\ \mathfrak{x} < \mathcal{p}_1, \mathfrak{x} \geq \mathcal{p}_5 \\ \sigma_{\tilde{\mathcal{P}}} \frac{(\mathfrak{x}-\mathcal{p}_1)}{(\mathcal{p}_2-\mathcal{p}_1)} & ;\ for\ \mathcal{p}_1 \leq \mathfrak{x} \leq \mathcal{p}_2 \\ \sigma_{\tilde{\mathcal{P}}} \frac{(\mathfrak{x}-\mathcal{p}_2)}{(\mathcal{p}_3-\mathcal{p}_2)} & ;\ for\ \mathcal{p}_2 \leq \mathfrak{x} \leq \mathcal{p}_3 \\ \sigma_{\tilde{\mathcal{P}}} & ;\ for\ \mathfrak{x} = \mathcal{p}_3 \\ \sigma_{\tilde{\mathcal{P}}} \frac{(\mathcal{p}_4-\mathfrak{x})}{(\mathcal{p}_4-\mathcal{p}_3)} & ;\ for\ \mathcal{p}_3 \leq \mathfrak{x} \leq \mathcal{p}_4 \\ \sigma_{\tilde{\mathcal{P}}} \frac{(\mathcal{p}_5-\mathfrak{x})}{(\mathcal{p}_5-\mathcal{p}_4)} & ;\ for\ \mathcal{p}_4 \leq \mathfrak{x} \leq \mathcal{p}_5 \end{cases} \quad (6)$$

and

$$\mathfrak{F}_{\tilde{\mathcal{P}}}(\mathfrak{x}) = \begin{cases} \tau_{\tilde{\mathcal{P}}} & ;\ for\ \mathfrak{x} < \mathcal{p}_1, \mathfrak{x} \geq \mathcal{p}_5 \\ \tau_{\tilde{\mathcal{P}}} \frac{(\mathcal{p}_2-\mathfrak{x})}{(\mathcal{p}_2-\mathcal{p}_1)} & ;\ for\ \mathcal{p}_1 \leq \mathfrak{x} \leq \mathcal{p}_2 \\ \tau_{\tilde{\mathcal{P}}} \frac{(\mathcal{p}_3-\mathfrak{x})}{(\mathcal{p}_3-\mathcal{p}_2)} & ;\ for\ \mathcal{p}_2 \leq \mathfrak{x} \leq \mathcal{p}_3 \\ 0 & ;\ for\ \mathfrak{x} = \mathcal{p}_3 \\ \tau_{\tilde{\mathcal{P}}} \frac{(\mathfrak{x}-\mathcal{p}_3)}{(\mathcal{p}_4-\mathcal{p}_3)} & ;\ for\ \mathcal{p}_3 \leq \mathfrak{x} \leq \mathcal{p}_4 \\ \tau_{\tilde{\mathcal{P}}} \frac{(\mathfrak{x}-\mathcal{p}_4)}{(\mathcal{p}_5-\mathcal{p}_4)} & ;\ for\ \mathcal{p}_4 \leq \mathfrak{x} \leq \mathcal{p}_5 \end{cases} \quad (7)$$

where $\sigma_{\tilde{\mathcal{P}}}, \tau_{\tilde{\mathcal{P}}} \in [0,1]$, $\mathcal{p}_1, \mathcal{p}_2, \mathcal{p}_3, \mathcal{p}_4, \mathcal{p}_5 \in \mathbb{R}$ and $0 \leq \mathfrak{T}_{\tilde{\mathcal{P}}}(\mathfrak{x}) + \mathfrak{F}_{\tilde{\mathcal{P}}}(\mathfrak{x}) \leq 1$.

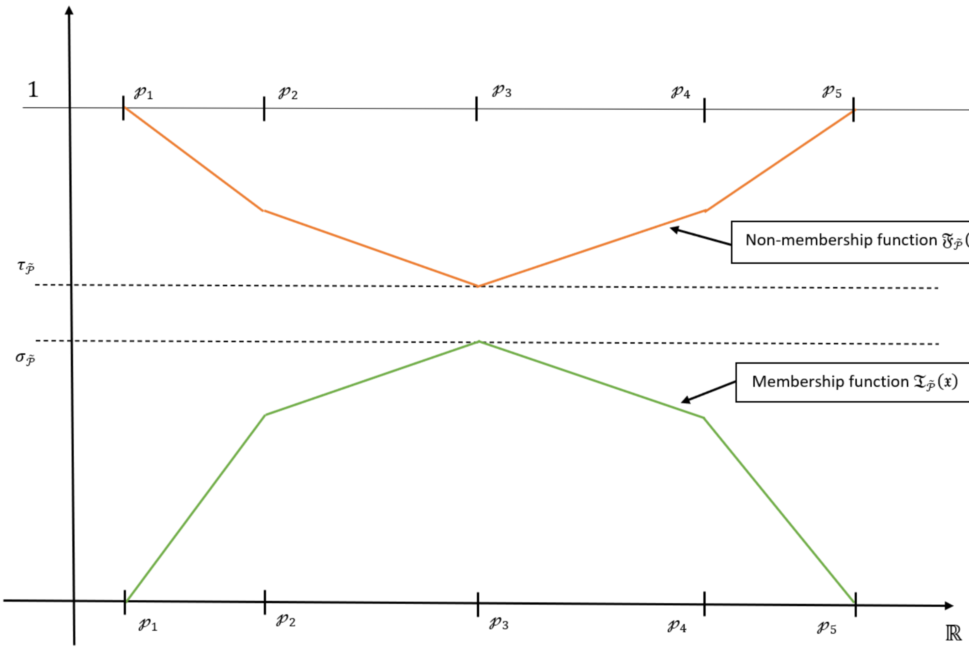

Fig. 1. Graphical representation of GPIFNs

Figure 1 shows the geometric representation of generalized pentagonal intuitionistic fuzzy numbers (GPFINs), where $\sigma_{\tilde{\mathcal{P}}}$ be the maximum membership value $\left(\mathfrak{T}_{\tilde{\mathcal{P}}}(\mathfrak{x})\right)$ and $\tau_{\tilde{\mathcal{P}}}$ be the minimum non-membership value $\left(\mathfrak{F}_{\tilde{\mathcal{P}}}(\mathfrak{x})\right)$ at the point $\mathcal{p}_3$.

3.3 Some arithmetic operations on generalized pentagonal intuitionistic fuzzy numbers

Let $\tilde{\mathcal{U}} = \langle(\mathcal{u}_1, \mathcal{u}_2, \mathcal{u}_3, \mathcal{u}_4, \mathcal{u}_5), \sigma_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{U}}}\rangle$ and $\tilde{\mathcal{V}} = \langle(\mathcal{v}_1, \mathcal{v}_2, \mathcal{v}_3, \mathcal{v}_4, \mathcal{v}_5), \sigma_{\tilde{\mathcal{V}}}, \tau_{\tilde{\mathcal{V}}}\rangle$ be two GPFINs. Then the arithmetic operations on GPFINs are as follows:

(i) Addition of two GPFINs:

$$\tilde{\mathcal{U}} + \tilde{\mathcal{V}} = \langle (u_1, u_2, u_3, u_4, u_5), \sigma_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{U}}} \rangle + \langle (v_1, v_2, v_3, v_4, v_5), \sigma_{\tilde{\mathcal{V}}}, \tau_{\tilde{\mathcal{V}}} \rangle$$
$$= \langle (u_1 + v_1, u_2 + v_2, u_3 + v_3, u_4 + v_4, u_5 + v_5), min\{\sigma_{\tilde{\mathcal{U}}}, \sigma_{\tilde{\mathcal{V}}}\}, max\{\tau_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{V}}}\} \rangle \quad (8)$$

(ii) Subtraction of GPFINs:

$$\tilde{\mathcal{U}} - \tilde{\mathcal{V}} = \langle (u_1, u_2, u_3, u_4, u_5), \sigma_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{U}}} \rangle - \langle (v_1, v_2, v_3, v_4, v_5), \sigma_{\tilde{\mathcal{V}}}, \tau_{\tilde{\mathcal{V}}} \rangle$$
$$= \langle (u_1 - v_5, u_2 - v_4, u_3 - v_3, u_4 - v_2, u_5 - v_1), min\{\sigma_{\tilde{\mathcal{U}}}, \sigma_{\tilde{\mathcal{V}}}\}, max\{\tau_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{V}}}\} \rangle \quad (9)$$

(iii) Scalar Multiplication of GPFIN:

$$\kappa\tilde{\mathcal{U}} = \kappa \times \tilde{\mathcal{U}}$$
$$= \begin{cases} \langle (\kappa u_1, \kappa u_2, \kappa u_3, \kappa u_4, \kappa u_5), \sigma_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{U}}} \rangle ; \text{ where } \kappa(\geq 0) \text{ is a positive number} \\ \langle (\kappa u_5, \kappa u_4, \kappa u_3, \kappa u_2, \kappa u_1), \tau_{\tilde{\mathcal{U}}}, \sigma_{\tilde{\mathcal{U}}} \rangle ; \text{ where } \kappa(< 0) \text{is a negative number} \end{cases} \quad (10)$$

(iv) Multiplication of two GPFINs:

$$\tilde{\mathcal{U}} \times \tilde{\mathcal{V}} = \langle (u_1, u_2, u_3, u_4, u_5), \sigma_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{U}}} \rangle \times \langle (v_1, v_2, v_3, v_4, v_5), \sigma_{\tilde{\mathcal{V}}}, \tau_{\tilde{\mathcal{V}}} \rangle$$
$$= \langle (u_1 v_1, u_2 v_2, u_3 v_3, u_4 v_4, u_5 v_5), min\{\sigma_{\tilde{\mathcal{U}}}, \sigma_{\tilde{\mathcal{V}}}\}, max\{\tau_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{V}}}\} \rangle \quad (11)$$

(v) Division of two GPFINs:

$$\frac{\tilde{\mathcal{U}}}{\tilde{\mathcal{V}}} = \langle \left(\frac{u_1}{v_5}, \frac{u_2}{v_4}, \frac{u_3}{v_3}, \frac{u_4}{v_2}, \frac{u_5}{v_1} \right), min\{\sigma_{\tilde{\mathcal{U}}}, \sigma_{\tilde{\mathcal{V}}}\}, max\{\tau_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{V}}}\} \rangle \quad (12)$$

for $u_5 > 0$ and $v_5 > 0$, respectively.

3.4 Score and accuracy functions

Order relation is not defined for fuzzy numbers. So, for making a comparison between two fuzzy numbers, the Score and Accuracy functions are used. Let $\tilde{\mathcal{U}} = \langle (u_1, u_2, u_3, u_4, u_5), \sigma_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{U}}} \rangle$ be a GPFIN. The Score function $\mathcal{S}_{\mathfrak{f}}(\tilde{\mathcal{U}})$ and the Accuracy function $\mathcal{A}_{\mathfrak{f}}(\tilde{\mathcal{U}})$ are defined as follows

$$\mathcal{S}_{\mathfrak{f}}(\tilde{\mathcal{U}}) = \frac{(u_1 + u_2 + u_3 + u_4 + u_5)(\sigma_{\tilde{\mathcal{U}}} - \tau_{\tilde{\mathcal{U}}})}{5} \quad (13)$$

$$\mathcal{A}_{\mathfrak{f}}(\tilde{\mathcal{U}}) = \frac{(u_1 + u_2 + u_3 + u_4 + u_5)(1 + \sigma_{\tilde{\mathcal{U}}} - \tau_{\tilde{\mathcal{U}}})}{10} \quad (14)$$

Theorem 1. Let $\tilde{\mathcal{U}} = \langle (u_1, u_2, u_3, u_4, u_5), \sigma_{\tilde{\mathcal{U}}}, \tau_{\tilde{\mathcal{U}}} \rangle$ and $\tilde{\mathcal{V}} = \langle (v_1, v_2, v_3, v_4, v_5), \sigma_{\tilde{\mathcal{V}}}, \tau_{\tilde{\mathcal{V}}} \rangle$ be two GPFINs. Then

[1]. If $\mathcal{S}_{\mathfrak{f}}(\tilde{\mathcal{U}}) \succ \mathcal{S}_{\mathfrak{f}}(\tilde{\mathcal{V}}) \Rightarrow \tilde{\mathcal{U}} \succ \tilde{\mathcal{V}}$;
[2]. If $\mathcal{S}_{\mathfrak{f}}(\tilde{\mathcal{U}}) \prec \mathcal{S}_{\mathfrak{f}}(\tilde{\mathcal{V}}) \Rightarrow \tilde{\mathcal{U}} \prec \tilde{\mathcal{V}}$;
[3]. If $\mathcal{S}_{\mathfrak{f}}(\tilde{\mathcal{U}}) = \mathcal{S}_{\mathfrak{f}}(\tilde{\mathcal{V}})$, then
  (a) $\mathcal{A}_{\mathfrak{f}}(\tilde{\mathcal{U}}) \succ \mathcal{A}_{\mathfrak{f}}(\tilde{\mathcal{V}}) \Rightarrow \tilde{\mathcal{U}} \succ \tilde{\mathcal{V}}$;
  (b) $\mathcal{A}_{\mathfrak{f}}(\tilde{\mathcal{U}}) \prec \mathcal{A}_{\mathfrak{f}}(\tilde{\mathcal{V}}) \Rightarrow \tilde{\mathcal{U}} \prec \tilde{\mathcal{V}}$;
  (c) $\mathcal{A}_{\mathfrak{f}}(\tilde{\mathcal{U}}) = \mathcal{A}_{\mathfrak{f}}(\tilde{\mathcal{V}}) \Rightarrow \tilde{\mathcal{U}} \sim \tilde{\mathcal{V}}$.

4. Proposed MCDM methodologies

In this section, we will discuss the MCDM methodologies which we will use in this paper. Multi-Criteria Decision Making (MCDM) is a well-known technique for handling complex decision-making problems. There are a variety of MCDM methods, such as AHP, CRITIC, Entropy [25], TOPSIS, VIKOR [26,27], MARCOS, etc. Some of them are used to evaluate the weight of criteria and the remaining are applied to get the ranking of the considered alternatives. In this problem, we have considered the Entropy method for weight evaluation and the VIKOR method for ranking alternatives.

4.1 Entropy weight method

The entropy weighted method was introduced by C. E. Shannon [25] in 1948. This is one of the most popular decision-making methods used to evaluate the weights of the criteria. This method [26] can determine the weight of each criterion in a problem with data uncertainty and multiple conflicting criteria. The step-by-step description of the Entropy method [28] is discussed as follows:

Let us consider $\mathcal{P}$ number of criteria such as $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \dots, \mathcal{C}_{\mathcal{P}}$ and $\mathcal{Q}$ number of alternatives, namely $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \dots, \mathcal{A}_{\mathcal{Q}}$. There are a total $\mathfrak{R}$ number of decision experts considered, for their viewpoint, decision matrices are formulated. The entropy methodology is formulated as follows:

A. *Formation of Decision Matrices* $(\tilde{\mathcal{G}}_\delta)$*:* Depending on the opinions of decision experts, total $\mathfrak{R}$ number of decision matrices $(\tilde{\mathcal{G}}_\delta)$ are formulated. The decision matrix is constructed based on δ^{th} decision expert, denoted by $\tilde{\mathcal{G}}_\delta$ and formulated in $\mathcal{Q} \times \mathcal{P}$ order. The δ^{th} decision matrix $\tilde{\mathcal{G}}_\delta$ defined as

$$\tilde{\mathcal{G}}_\delta = \left[(\tilde{\mathcal{U}}_{mn})_\delta\right]_{\mathcal{Q}\times\mathcal{P}} = \begin{bmatrix} (\tilde{\mathcal{U}}_{11})_\delta & (\tilde{\mathcal{U}}_{12})_\delta & \dots & (\tilde{\mathcal{U}}_{1n})_\delta & \dots & (\tilde{\mathcal{U}}_{1\mathcal{P}})_\delta \\ (\tilde{\mathcal{U}}_{21})_\delta & (\tilde{\mathcal{U}}_{22})_\delta & \dots & (\tilde{\mathcal{U}}_{2n})_\delta & \dots & (\tilde{\mathcal{U}}_{2\mathcal{P}})_\delta \\ \vdots & \vdots & \ddots & \dots & \ddots & \vdots \\ (\tilde{\mathcal{U}}_{m1})_\delta & (\tilde{\mathcal{U}}_{m2})_\delta & \dots & (\tilde{\mathcal{U}}_{mn})_\delta & \dots & (\tilde{\mathcal{U}}_{m\mathcal{P}})_\delta \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ (\tilde{\mathcal{U}}_{\mathcal{Q}1})_\delta & (\tilde{\mathcal{U}}_{\mathcal{Q}2})_\delta & \dots & (\tilde{\mathcal{U}}_{\mathcal{Q}n})_\delta & \dots & (\tilde{\mathcal{U}}_{\mathcal{Q}\mathcal{P}})_\delta \end{bmatrix}_{\mathcal{Q}\times\mathcal{P}} \quad (15)$$

where $\delta = 1,2,\dots,\mathfrak{R}$, $m = 1,2,\dots,\mathcal{Q}$, $n = 1,2,\dots,\mathcal{P}$ and each entry $(\tilde{\mathcal{U}}_{mn})_\delta$ is of the form

$$(\tilde{\mathcal{U}}_{mn})_\delta = \langle(((\wp_1)_{mn})_\delta, ((\wp_2)_{mn})_\delta, ((\wp_3)_{mn})_\delta, ((\wp_4)_{mn})_\delta, ((\wp_5)_{mn})_\delta), \sigma_{(mn)_\delta}, \tau_{(mn)_\delta}\rangle \quad (16)$$

B. *Construction of Aggregated Decision Matrix* $(\tilde{\mathcal{G}}_{\mathfrak{A}})$*:* In this step, these decision matrices $(\tilde{\mathcal{G}}_\delta)$ are aggregated into a single decision matrix by using Equation (17), as follows

$$\begin{cases} (\wp_1)_{mn} = \min_{\delta=1,2,\dots,\mathfrak{R}} ((\wp_1)_{mn})_\delta \\ (\wp_2)_{mn} = \min_{\delta=1,2,\dots,\mathfrak{R}} ((\wp_2)_{mn})_\delta \\ (\wp_3)_{mn} = \sqrt[\mathfrak{R}]{\prod_{\delta=1}^{\mathfrak{R}} ((\wp_1)_{mn})_\delta} \\ (\wp_4)_{mn} = \max_{\delta=1,2,\dots,\mathfrak{R}} ((\wp_4)_{mn})_\delta \\ (\wp_5)_{mn} = \max_{\delta=1,2,\dots,\mathfrak{R}} ((\wp_5)_{mn})_\delta \\ \sigma_{(\tilde{\mathcal{U}}_{mn})} = \min_{\delta=1,2,\dots,\mathfrak{R}} \sigma_{(\tilde{\mathcal{U}}_{mn})_\delta} \\ \tau_{(\tilde{\mathcal{U}}_{mn})} = \max_{\delta=1,2,\dots,\mathfrak{R}} \tau_{(\tilde{\mathcal{U}}_{mn})_\delta} \end{cases} \quad (17)$$

where $\delta = 1,2,\dots,\mathfrak{R}$, $m = 1,2,\dots,\mathcal{Q}$ and $n = 1,2,\dots,\mathcal{P}$. The Aggregated decision matrix $(\tilde{\mathcal{G}}_{\mathfrak{A}})$ takes the form as $\tilde{\mathcal{G}}_{\mathfrak{A}} = \left[(\tilde{\mathcal{U}}_{mn})\right]_{\mathcal{Q}\times\mathcal{P}}$, where

$$\tilde{\mathcal{U}}_{mn} = \langle((\wp_1)_{mn}, (\wp_2)_{mn}, (\wp_3)_{mn}, (\wp_4)_{mn}, (\wp_5)_{mn}), \sigma_{(\tilde{\mathcal{U}}_{mn})}, \tau_{(\tilde{\mathcal{U}}_{mn})}\rangle \quad (18)$$

C. *Normalization of the Decision Matrix* $(\tilde{\mathcal{G}}_{\mathcal{N}})$*:* In this step normalization of the aggregated decision matrix $(\tilde{\mathcal{G}}_{\mathfrak{A}})$ is performed and it is denoted by $\tilde{\mathcal{G}}_{\mathcal{N}} = [(\tilde{\mathfrak{r}}_{mn})]_{\mathcal{Q}\times\mathcal{P}}$, where

$$\tilde{\mathfrak{r}}_{mn} = \langle\left(\frac{(\wp_1)_{mn}}{(\wp_5)_n^+}, \frac{(\wp_2)_{mn}}{(\wp_5)_n^+}, \frac{(\wp_3)_{mn}}{(\wp_5)_n^+}, \frac{(\wp_4)_{mn}}{(\wp_5)_n^+}, \frac{(\wp_5)_{mn}}{(\wp_5)_n^+}\right), \sigma_n, \tau_n\rangle \quad (19)$$

and $(\wp_5)_n^+ = \max_m (\wp_5)_{mn}$ for the beneficial criterion (n) and

$$\tilde{\mathfrak{r}}_{mn} =< \left(\frac{(\wp_1)_n^-}{(\wp_5)_{mn}}, \frac{(\wp_1)_n^-}{(\wp_4)_{mn}}, \frac{(\wp_1)_n^-}{(\wp_3)_{mn}}, \frac{(\wp_1)_n^-}{(\wp_2)_{mn}}, \frac{(\wp_1)_n^-}{(\wp_1)_{mn}}\right), \sigma_n, \ \tau_n > \quad (20)$$

and $(\wp_1)_n^- = \min_m (\wp_5)_{mn}$ for non-beneficial criterion (n).

D. *Score Valued Normalised Matrix* $(\mathcal{G}_s)$*:* In this step, each entry of the normalized decision matrix $(\mathcal{G}_s)$, which is given in the form of Generalised Pentagonal Intuitionistic Fuzzy

Number are converted to an equivalent crisp value by evaluating it's score value by using Equation (13). The Score valued normalised matrix can be written as, $\mathcal{G}_{s} = [\mathcal{r}_{mn}]_{\mathcal{Q}\times\mathcal{P}}$, where $\mathcal{r}_{mn}$ is the score value of $\tilde{\mathfrak{r}}_{mn}$.

E. *Calculating Entropy Value* $(\mathcal{E}_n)$*:* In this step, the Entropy value $(\mathcal{E}_n)$ for each criterion (n) will be evaluated by using the following formula

$$\mathcal{E}_n = -\frac{1}{log\mathcal{Q}} \times \sum_{m=1}^{\mathcal{Q}}\{\mathcal{r}_{mn} \times log(\mathcal{r}_{mn})\} \quad (21)$$

where $m = 1,2, \dots, \mathcal{Q}$ and $n = 1,2, \dots, \mathcal{P}$.

F. *Evaluating Degree of Dissatisfaction* $(\mathfrak{D}_n)$*:* The degree of dissatisfaction $(\mathfrak{D}_n)$ is evaluated in this step with the help of the following formula

$$\mathfrak{D}_n = (1 - \mathcal{E}_n) \quad (22)$$

for $n = 1,2, \dots, \mathcal{P}$.

G. *Calculating Criteria Weight* $(\mathcal{W}_n)$*:* The weight of each criterion $(\mathcal{W}_n)$ is evaluated by using the following formula

$$\mathcal{W}_n = \frac{\mathfrak{D}_n}{\sum_{n=1}^{\mathcal{P}} \mathfrak{D}_n} \quad (23)$$

where $n = 1,2, \dots, \mathcal{P}$. Equation (23) shows the weights $(\mathcal{W}_n)$ of the criteria (n) using the entropy-weighted method. The flowchart for the Entropy method's steps is shown in Figure 2.

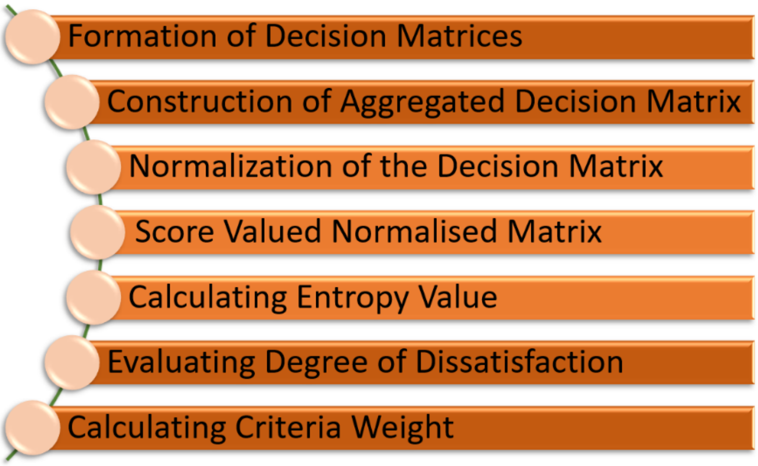

Fig. 2. Flowchart of the Entropy Method

4.2 VIKOR method

The Vlse Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method [26,29] is widely used method for ranking alternatives. The aim of this method is to find a feasible solution which is as close to the ideal solution as possible. The process of the VIKOR method [27] is discussed as follows.

Let us consider $\mathcal{P}$ number of criteria such as $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \dots, \mathcal{C}_{\mathcal{P}}$ and $\mathcal{Q}$ number of alternatives, namely $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \dots, \mathcal{A}_{\mathcal{Q}}$. There is total $\mathfrak{R}$ number of decision experts depends on the viewpoint from them the decision matrices are formulated. For alternative $\mathcal{A}_m$, the related rating of n^{th} criterion given by the δ^{th} decision expert is represented as $\left(\tilde{\mathcal{U}}_{mn}\right)_{\delta}$, where $\delta = 1,2, \dots, \mathfrak{R}$, $m = 1,2, \dots, \mathcal{Q}$ and $n = 1,2, \dots, \mathcal{P}$. The computational steps of the VIKOR methodology are as follows:

I. *Formation of Decision Matrices* $(\tilde{\mathcal{G}}_{\delta})$*:* This method is the same as we have mentioned in the first step of the Entropy method and the decision matrices $(\tilde{\mathcal{G}}_{\delta})$ are of the form

$$\tilde{\mathcal{G}}_{\delta} = \left[\left(\tilde{\mathcal{U}}_{mn}\right)_{\delta}\right]_{\mathcal{Q}\times\mathcal{P}} \quad (24)$$

where $\delta = 1,2,\dots,\mathfrak{R}$, $m = 1,2,\dots,\mathcal{Q}$ and $n = 1,2,\dots,\mathcal{P}$.

II. *Construction of Aggregated Decision Matrix* $\left(\tilde{\mathcal{G}}_{\mathfrak{A}}\right)$*:* In this step, we also proceed in the same way as we mentioned in the second step of the Entropy method. The aggregated decision matrix takes the form of

$$\tilde{\mathcal{G}}_{\mathfrak{A}} = \left[\left(\tilde{\mathcal{U}}_{mn}\right)\right]_{\mathcal{Q}\times\mathcal{P}} \tag{25}$$

where $\tilde{\mathcal{U}}_{mn} = \langle((\wp_1)_{mn},(\wp_2)_{mn},(\wp_3)_{mn},(\wp_4)_{mn},(\wp_5)_{mn}),\sigma_{(\tilde{\mathcal{U}}_{mn})},\tau_{(\tilde{\mathcal{U}}_{mn})}\rangle$.

III. *Finding the Best Value* $\left(\tilde{\mathcal{U}}_n^+\right)$ *and Worst Value* $\left(\tilde{\mathcal{U}}_n^-\right)$*:* As there are some beneficial and some non-beneficial criteria, so the best value $\left(\tilde{\mathcal{U}}_n^+\right)$ and worst value $\left(\tilde{\mathcal{U}}_n^-\right)$, of all criteria (n) are evaluated as follows

a. If n^{th} criteria is beneficial criteria, then

$$\begin{cases}\tilde{\mathcal{U}}_n^+ = \max\limits_m \tilde{\mathcal{U}}_{mn}\\ \tilde{\mathcal{U}}_n^- = \min\limits_m \tilde{\mathcal{U}}_{mn}\end{cases} \tag{26}$$

b. If n^{th} criterion is non-beneficial criteria, then

$$\begin{cases}\tilde{\mathcal{U}}_n^+ = \min\limits_m \tilde{\mathcal{U}}_{mn}\\ \tilde{\mathcal{U}}_n^- = \max\limits_m \tilde{\mathcal{U}}_{mn}\end{cases} \tag{27}$$

where $m = 1,2,\dots,\mathcal{Q}$ and $n = 1,2,\dots,\mathcal{P}$.

IV. *Construction of Score Valued Modified Aggregated Matrix* $(\mathcal{G}_{\mathfrak{d}})$*:* In this step, the score value of each entry of the aggregated decision matrix, along with the best value $\left(\tilde{\mathcal{U}}_n^+\right)$ and worst value $\left(\tilde{\mathcal{U}}_n^-\right)$ are evaluated by using Equation (13). The required matrix is denoted by

$$\mathcal{G}_{\mathfrak{d}} = [(\mathcal{U}_{mn})]_{(\mathcal{Q}+2)\times\mathcal{P}} \tag{28}$$

where $\mathcal{U}_{mn}$ is the score value corresponding to the generalised pentagonal intuitionistic fuzzy number (GPIFN) $\tilde{\mathcal{U}}_{mn}$.

V. *Evaluating the value of* $\mathcal{S}_m$ *&* $\mathcal{R}_m$*:* The value of $\mathcal{S}_m$ & $\mathcal{R}_m$ are evaluated by using the following formula

$$\mathcal{S}_m = \sum_{n=1}^{\mathcal{P}}\left(\mathcal{W}_n \times \frac{(\mathcal{U}_n^+ - \mathcal{U}_{mn})}{(\mathcal{U}_n^+ - \mathcal{U}_n^-)}\right) \tag{29}$$

$$\mathcal{R}_m = \max_m\left\{\mathcal{W}_n \times \frac{(\mathcal{U}_n^+ - \mathcal{U}_{mn})}{(\mathcal{U}_n^+ - \mathcal{U}_n^-)}\right\} \tag{30}$$

where $m = 1,2,\dots,\mathcal{Q}$ and $n = 1,2,\dots,\mathcal{P}$.

VI. *Calculation of VIKOR Index* $(\mathcal{Q}_m)$*:* VIKOR Index $(\mathcal{Q}_m)$ for each alternative (m) is calculated by using the following equation

$$\mathcal{Q}_m = \gamma\frac{(\mathcal{S}_m - \mathcal{S}^*)}{(\mathcal{S}^- - \mathcal{S}^*)} + (1-\gamma)\frac{(\mathcal{R}_m - \mathcal{R}^*)}{(\mathcal{R}^- - \mathcal{R}^*)} \tag{31}$$

where $\mathcal{S}^* = \min\limits_m \mathcal{S}_m$, $\mathcal{S}^- = \max\limits_m \mathcal{S}_m$ and $\mathcal{R}^* = \min\limits_m \mathcal{R}_m$, $\mathcal{R}^- = \max\limits_m \mathcal{R}_m$; where $\gamma \in [0,1]$ is the weight of the decision-making strategy of the majority of attributes.

VII. *Ranking of Alternatives:* Depending on the values of $\mathcal{Q}_m$ the ranking of alternatives is performed. An alternative gets the 1[st] rank if it possesses the lowest value of $\mathcal{Q}_m$ and so on, which means an alternative having a minimum value of $\mathcal{Q}_m$ will get the better rank. The flowchart of the VIKOR method is shown in Figure 3.

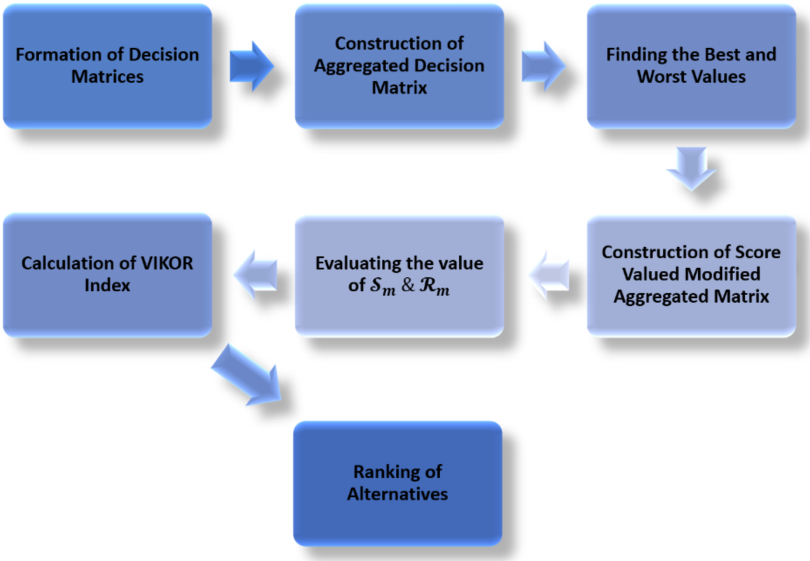

Fig. 3. Flowchart of the VIKOR Method

5. Criteria and alternative selection

In this section, we will discuss the criteria which are essential to find the ranking of higher educational institutions. Further, we are representing the details of alternatives for ranking different institutions.

5.1 Criteria selection

Students take admission in higher educational institutions to continue their higher studies and to improve their employability in the future. There are many higher educational institutions, so choosing a particular higher educational institution as the best one is a very difficult task. There are many facts on which the assessment of the ranking of an institution is dependent. After going through many research articles, we have selected 7 criteria for this purpose. They are discussed as follows:

A. Quality of Teaching and Learning ($\mathcal{C}_1$):

Quality of Teaching and Learning [30-32] is a crucial criterion for the ranking of Higher Educational Institutions. Teaching is the process of delivering the knowledge and skills to students and learning means is how students can understand the concepts given by their teachers. The quality of teaching depends on some factors like availability of a sufficient number of qualified teachers, availability of learning resources, opportunity of integrating technology into classroom activities, etc. The teacher may incorporate new techniques for making the teaching-learning process more interesting and effective. Through improving the quality of teaching, it is possible to establish a student-centred learning atmosphere and to help students improve their critical thinking. An institution that focuses on maintaining the high quality of teaching and learning experiences can ensure better academic success and overall growth of students.

B. Research and Innovation ($\mathcal{C}_2$):

Research and Innovation [33,34] is an important criterion for assessing the ranking of Higher Educational Institutions. It reflects how much the students and faculty of an institute contribute to enrich the knowledge, modify and improve the technologies and develop the society through their new inventions and research works. This criterion evaluates the number of research papers, the quality of research articles, the number of patents and sponsored projects, etc. It also focuses on the facts that whether an institute is working with collaboration with other research organization and industrial groups. It also considers the institution’s approach to encourage students and faculties to work on new innovative ideas through providing them with the facilities of using well-equipped labs and research centres. This research and innovations not only enhance the reputation of an institute but also help to resolve real-life problems.

C. Employability and Industry Linkages ($\mathcal{C}_3$)*:*

Employability and Industry Linkages [35,36] are a vital criterion in the process of ranking Higher Educational Institutions. Depending on their future goals, students take admission in an institution and continue their higher education. Nowadays, to get a better opportunity in the job market, only theoretical knowledge and a higher degree are not enough. Employers always try to recruit people having good knowledge, along with skills and expertise related to their job position. So, it is important that higher educational institution moderate their curriculum according to the industrial need, provide opportunities to their students to develop the skills and participate in internship programs. Educational institutions having good linkages with many Industries can accelerate the opportunity of students for getting practical knowledge through training and employability.

D. Infrastructure, Learning Resources and Financial Support ($\mathcal{C}_4$)*:*

Infrastructure, Learning Resources and Financial support [37,38] are also important criteria for ranking higher educational institutions. For providing quality education, it is very important to focus on developing the infrastructure of the institution. Students will benefit if facilities for using well-maintained laboratories, libraries, smart classrooms and clean washrooms are given to them within the campus. To continue the research and innovative work, an ICT-enabled learning space, modern-equipped research labs and availability of learning resources are very much needed in a higher educational institution. A higher educational institution that has the capability to provide financial assistance through giving stipends to the meritorious students, research scholars and funding for working on various projects, can motivate new ideas of youngsters.

E. Expenses of Higher Studies($\mathcal{C}_5$)*:*

Expenses of Higher studies [39,40] play a vital role in the process of assessing the ranking of Higher Educational Institutions. Students coming from different places are always trying to take admission in a reputed higher educational institute to get a better future ahead. But sometimes the economic status of students becomes a barrier, as students coming from the economically weaker section are not able to bear the expenses of higher study. There is a lack of reputable institutions for pursuing higher studies in many places. So, the students belonging to those areas have to go to a distant location for their further study and they have to pay a large amount of money for their food and lodging. There are some government-sponsored educational institutions where students can get quality education and the course expenses are not so high there but the number of such institutions is not sufficient with respect to the population. Thus, it is very difficult for students from poor families to achieve higher studies in a reputed institute without any scholarship or financial support.

F. Governance and Administration ($\mathcal{C}_6$)*:*

Governance and Administration [41,42] is an important criterion for the process of ranking higher educational institutions. There are various types of functions to be performed by the administration to run the educational institute smoothly. An educational institute, along with good governance and administration, can work properly and execute the activities according to the educational calendar within the time period. It is an administration’s duty to make decisions regarding activities such as recruiting faculty, modifying the curriculum, organizing conferences and workshops and utilizing the funding for the betterment of the institution. There are many research and innovative works going on in higher educational institutions collaborating with some other institutions or industries that need a huge supply of funding, modern research labs with proper equipment, so there is a need for good governance of such works.

G. Socio-cultural Situation ($\mathcal{C}_7$):

Students who aim to pursue higher education dream of taking admission in an institution popular for its educational excellence. So, students coming from different locations try to take admission in a reputed institution. In a country like India, students are from different social and cultural backgrounds, so they have different beliefs. In many areas, there is a lack of social acceptance that females and girls can get equal opportunity in study, equal opportunity to empower and thus they face discrimination to pursue higher education. In many places, girls are forced into early marriages in the name of cultural rituals, so girls from those places face obstacles in achieving opportunities in higher education. In many places, parents don't want to let go of their children to a distant location for further study and force their children to get admission in their nearby institution, as they think their social and cultural beliefs will be conserved there. Thus, the socio-cultural situation [43,44] becomes an important criterion.

5.2. Alternative selection

In this paper, we have considered three Higher educational institutions, namely $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$ as alternatives. Here, we want to evaluate the ranking of these 3 alternatives depending on 7 criteria, which are mentioned in the above section.

6. Model construction and data collection

In this paper, our main aim is to design a framework for ranking higher educational institutions, including two well-known MCDM methods, such as Entropy and VIKOR methods, in a generalized pentagonal intuitionistic fuzzy numbers (GPFINs) environment. Here, we have included 7 criteria which are essential for determining the ranking of higher educational institutions and 3 educational institutions are considered as alternatives. The opinions of 3 decision experts are considered, which are provided in linguistic terms with the help of Table 1. The details of these decision experts are given as follows

DM1: A professor of the education department from a renowned educational institution.

DM2: A policymaker from the Ministry of Education.

DM3: An administrative officer from the educational department.

Table 1
Conversion table between linguistic terms and GPFIN

Linguistic Term	GPFIN	Score Value	Accuracy Value
Absolutely Important (AI)	< (9,9.2,9.5,9.8,10.0),0.90,0.05 >	8.1	8.8
Strongly Important (SI)	< (8.5,8.7,9.0,9.3,9.5),0.85,0.07 >	7.0	8.0
Very Important (VI)	< (8.0,8.3,8.5,8.8,9.0),0.80,0.10 >	6.0	7.2
Equally Important (EI)	< (7.5,7.7,8.0,8.3,8.5),0.75,0.13 >	5.0	6.5
Moderately Important (MI)	< (7.0,7.2,7.5,7.8,8.0),0.71,0.18 >	4.0	5.7
Poor Important (PI)	< (6.5,6.7,7.0,7.3,7.5),0.65,0.22 >	3.0	5.0
Less Important (LI)	< (6.0,6.2,6.5,6.8,7.0),0.61,0.28 >	2.1	4.3

Thus, three 3×7 decision matrices are formulated using linguistic terms. Further, we have converted the linguistic terms into equivalent generalized pentagonal intuitionistic fuzzy numbers (GPFINs) using Table 1. Three decision matrices are shown in Table 2, where each entry **i**s given in linguistic terms. These decision matrices are taken for further evaluations.

Table 2
Decision Matrices in linguistic terms between criteria & alternatives based on three DMs

	Criteria vs Alternative	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
DM1	$\mathcal{A}_1$	AI	SI	SI	VI	MI	SI	PI
	$\mathcal{A}_2$	SI	VI	VI	MI	EI	VI	LI
	$\mathcal{A}_3$	SI	SI	VI	EI	MI	AI	MI
	Criteria vs Alternative	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
DM2	$\mathcal{A}_1$	AI	AI	VI	EI	PI	SI	PI
	$\mathcal{A}_2$	VI	SI	EI	VI	MI	SI	PI
	$\mathcal{A}_3$	SI	VI	SI	MI	VI	VI	MI
	Criteria vs Alternative	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
DM3	$\mathcal{A}_1$	SI	AI	SI	VI	MI	VI	MI
	$\mathcal{A}_2$	VI	SI	EI	MI	MI	SI	PI
	$\mathcal{A}_3$	VI	SI	VI	EI	EI	SI	PI

7. Numerical illustration and discussion

The numerical calculations are discussed in this section. Here, we have applied the entropy method step-by-step as described in Subsection 4.1 to evaluate the weight of each criterion. At first, the three decision matrices, which are given in Table 2, are converted to equivalent generalized pentagonal intuitionistic fuzzy numbers (GPFINs) using Table 1. Then we aggregated the Decision matrices into a single decision matrix and normalized it. Further, we have calculated the Score value corresponding to each GPFIN with the help of Equation (13). Then, proceeding as mentioned in the entropy method, we have obtained the criteria weights, which are given in Table 3.

Table 3
Criteria weight with associated data calculated by the entropy method

Criteria	Weight
Quality of Teaching and Learning ($\mathcal{C}_1$)	0.241
Research and Innovation ($\mathcal{C}_2$)	0.242
Employability and Industry Linkages ($\mathcal{C}_3$)	0.182
Infrastructure, Learning Resources and Financial support ($\mathcal{C}_4$)	0.059
Expenses of Higher studies($\mathcal{C}_5$)	0.075
Governance and Administration ($\mathcal{C}_6$)	0.191
Socio-cultural Situation ($\mathcal{C}_7$)	0.010

From Table 3, we can conclude that Research and Innovation ($\mathcal{C}_2$) is the most important criterion as it has acquired the weight 0.242, which is the highest among all weights. Due to the same reason, the criteria Quality of Teaching and Learning ($\mathcal{C}_1$) and Governance and Administration ($\mathcal{C}_6$) become the second and third most important criteria, respectively.

Remarks 1: Each criterion, along with its weights, is represented in Figure 4. It helps to better understand with a visual impact.

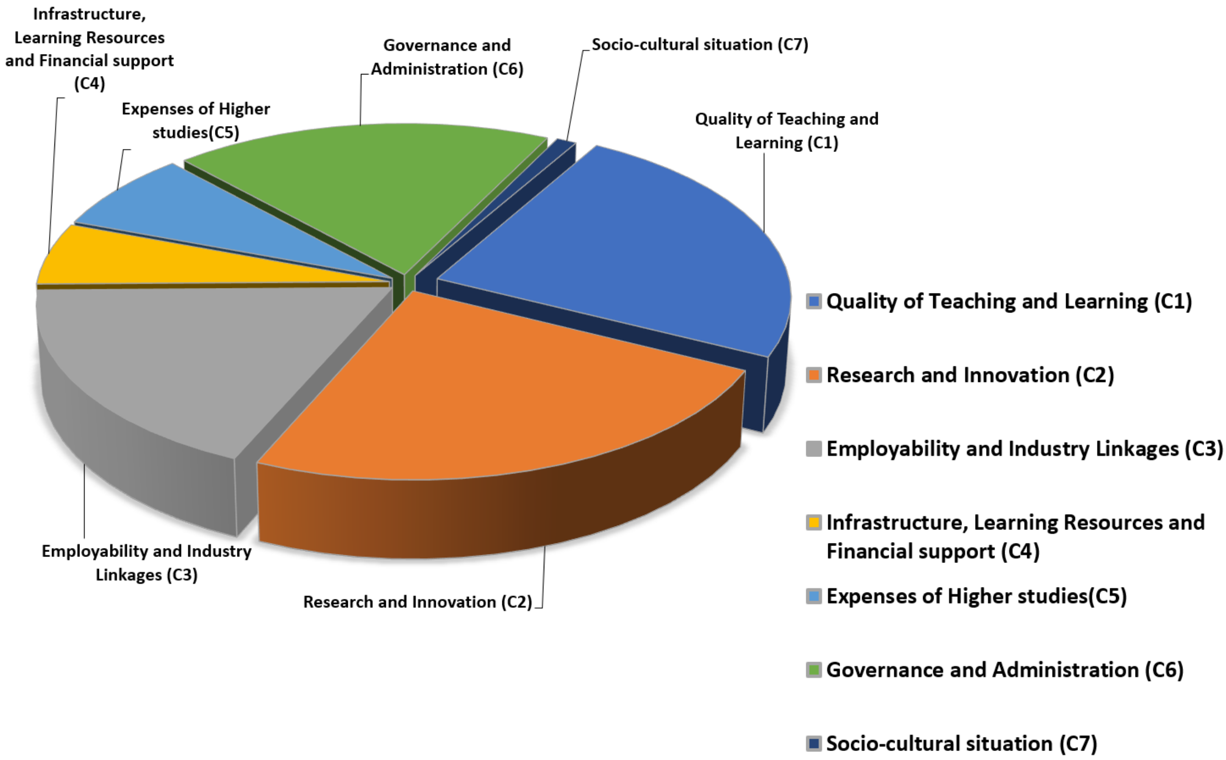

Fig. 4. Weight of the Criteria in the Pie structure evaluated by the Entropy method

We have used these weights in our further evaluation. Next, we have proceeded to find the ranking of alternatives by applying the VIKOR method. The decision matrices, which we have already evaluated for the entropy method, are used here and the Aggregated Decision matrix is formulated. Further, the best and worst values are calculated and then the score valued modified aggregated decision matrix is obtained by using Equation (13). Then the values of $\mathcal{S}_m$, $\mathcal{R}_m$ and VIKOR Index $(\mathcal{Q}_m)$ for each alternative, the evaluations are shown in Table 4.

Table 4
Ranking of the alternatives with $\mathcal{Q}_m$ value determined by the VIKOR methodology

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$	$\mathcal{S}_m$	$\mathcal{R}_m$	$\mathcal{Q}_m$	Rank
$\mathcal{A}_1$	-0.119	-0.125	-0.090	-0.117	-0.043	0.000	0.103	-0.392	0.103	0.00	1
$\mathcal{A}_2$	0.353	0.361	0.266	0.139	0.102	0.191	-0.089	1.323	0.361	1.00	3
$\mathcal{A}_3$	0.343	0.361	-0.082	0.169	0.123	0.000	0.106	1.020	0.361	0.91	2

From Table 4, it can be seen that the VIKOR Index is maximum for the alternative $\mathcal{A}_2$ and a minimum for the alternative $\mathcal{A}_1$. Thus, the alternative $\mathcal{A}_1$, $\mathcal{A}_3$, and $\mathcal{A}_2$ has obtained the 1st ranking, 2nd ranking and 3rd ranking, respectively. The ranking of these three alternatives can be visualized through the bar graph shown in Figure 5.

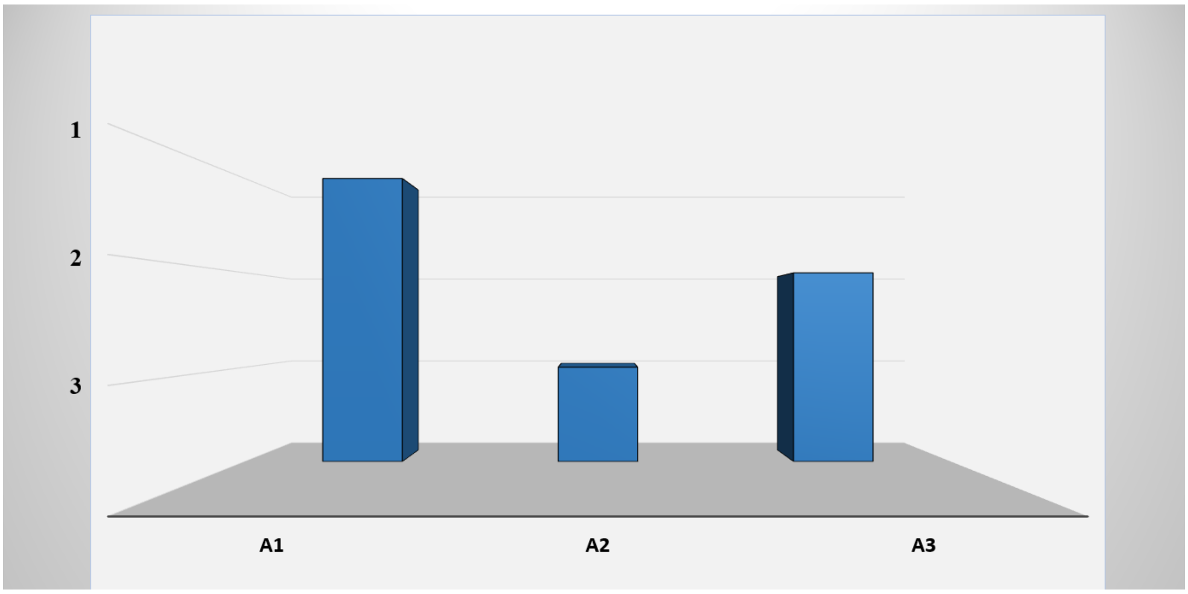

Fig. 5. Bar diagram of the alternatives based on Rank using VIKOR method

8. Sensitivity and comparative analysis

In this section, sensitivity analysis and comparative analysis are performed to check the stability and robustness of the obtained results, respectively.

8.1 Sensitivity analysis

Sensitivity analysis is used to assess the impact of small changes in input data on the final result. Thus, it helps to determine how the result is stable under slightly changed input data. Here, we have considered 3 cases for sensitivity Analysis, which are discussed as follows

Case 1: Eliminating the criterion Infrastructure, Learning Resources and Financial Support $(\mathcal{C}_4)$*:* We have eliminated the criteria Infrastructure, Learning Resources and Financial support $(\mathcal{C}_4)$ to check its effect on the ranking of higher educational institutions. From this Entropy-VIKOR method-based framework, we have obtained that the ranking of alternatives remains unaltered after omitting the criterion Infrastructure, Learning Resources and Financial support $(\mathcal{C}_4)$.

Case 2: Eliminating the Criterion Quality of Teaching and Learning $(\mathcal{C}_1)$*:* The Quality of Teaching and Learning $(\mathcal{C}_1)$ is one of the most important criteria for ranking higher educational institutions. After applying the Entropy method, it is known that it has acquired the 2nd highest weight among all criteria. Now, after omitting this criterion from this Entropy-VIKOR-based framework, we have evaluated the ranking of alternatives, which remains the same as the previous result. Thus, the elimination of the criterion Quality of Teaching and Learning $(\mathcal{C}_1)$ is not able to change the ranking of alternatives.

Case 3: Interchanging the weights of Employability and Industry Linkages $(\mathcal{C}_3)$ *and Governance and Administration* $(\mathcal{C}_6)$*:* Here we have interchanged the weights of the criteria, Employability and Industry Linkages $(\mathcal{C}_3)$ and Governance and Administration $(\mathcal{C}_6)$ to check how the result will be affected through this change. After applying the VIKOR method, we have obtained that the ranking of alternatives remains unchanged, which shows the stability of the result.

The results of three cases, along with the original ranking of alternatives $\mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_3$ are shown in Table 5.

Table 5
Sensitivity Analysis of the ranking of the higher educational institution model

Alternative	Case 1	Case 2	Case 3	Rank
$\mathcal{A}_1$	1	1	1	1
$\mathcal{A}_2$	3	3	3	3
$\mathcal{A}_3$	2	2	2	2

Remarks 2: Ranking of alternatives in three different cases under the entropy-VIKOR methodology and the proposed method are presented in Table 5. Additionally, the graphical representation of sensitivity analysis is shown in Figure 6.

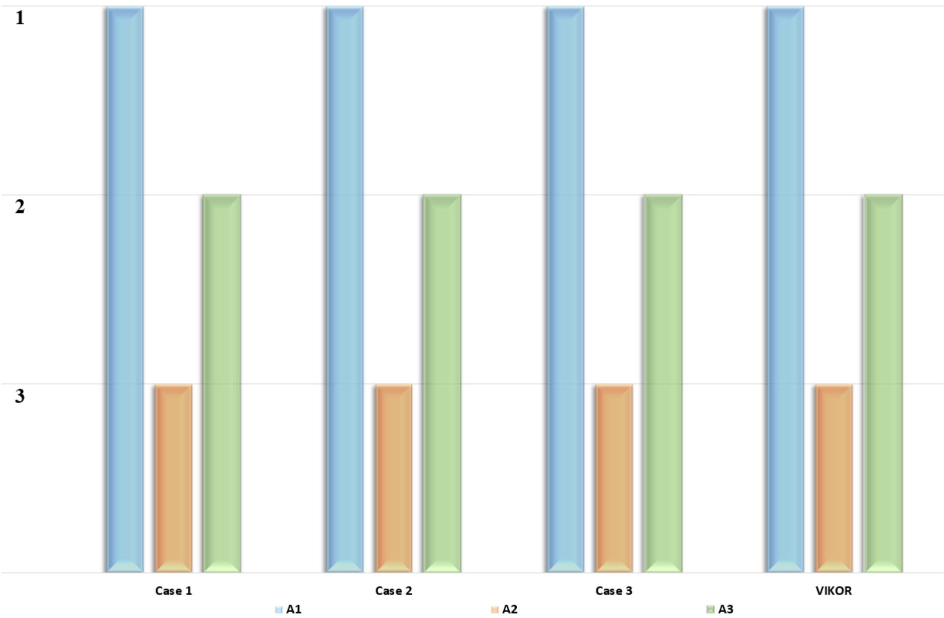

Fig. 6. Bar diagram of the alternatives under sensitivity analysis using the entropy-VIKOR method

8.2 Comparative analysis

In this subsection, we will discuss about the Comparison analysis. Comparative analysis helps us to check the robustness of the result. Here, we have considered two MCDM methods, namely TOPSIS and COPRAS, to determine the ranking of alternatives. Furthermore, the evaluated results are compared with the results obtained from the TOPSIS and COPRAS methods. After performing these two MCDM methods, we have obtained that the ranking of alternatives is the same as that we obtained by applying the VIKOR method. The ranking of alternatives in different MCDM methods is shown in Table 6.

Table 6
Comparative Analysis of the ranking of higher educational institution models

Alternative	VIKOR	TOPSIS	COPRAS
$\mathcal{A}_1$	1	1	1
$\mathcal{A}_2$	3	3	3
$\mathcal{A}_3$	2	2	2

Remarks 3: Ranking of alternatives in three different MCDM methods which are given in Table 6 are graphically represented in Figure 7.

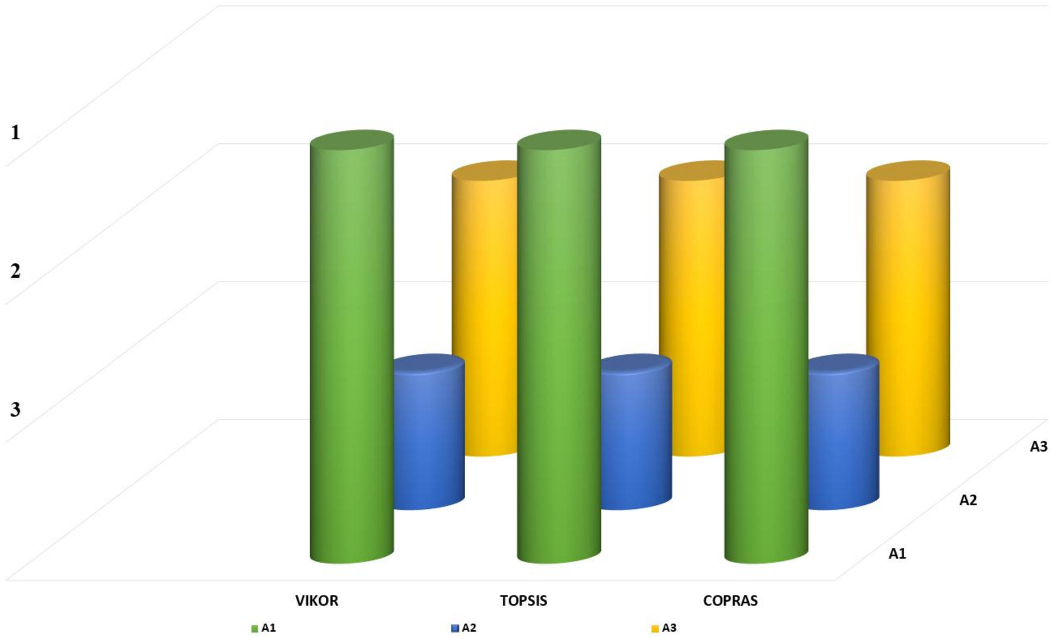

Fig. 7. Comparative analysis of the alternative ranking among VIKOR, COPRAS & TOPSIS methodologies

9. Conclusions

For the advancement of higher education, it is necessary to focus on increasing the number of higher educational institutions equipped with all types of modern facilities. After completing the basic education, students dream of getting admission in one of the best higher educational institutions as per their requirements. In the process of choosing the best higher education institution among a huge number of options, the ranking of higher educational institutions becomes a concerning fact. Here, we have established an entropy-VIKOR-based model in a generalized pentagonal intuitionistic fuzzy environment for finding the rank of higher educational institutions. Here, we have considered 7 factors as criteria, some of which are beneficial and some are non-beneficial, along with 3 higher educational institutions as alternatives and the opinions of 3 decision experts are included as a dataset. After performing the entropy weighted method, we have obtained that Research and Innovation $(\mathcal{C}_2)$ is the most important criterion among all criteria for evaluating the ranking of alternatives and the criterion Socio-cultural Situation $(\mathcal{C}_7)$ acquired the lowest weightage in this decision-making problem. After that, we have applied the VIKOR method and the alternatives $\mathcal{A}_1$, $\mathcal{A}_3$ and $\mathcal{A}_2$ obtained the ranking in increasing order. Thus, the alternative $\mathcal{A}_1$ is the best higher educational institution among these 3 alternatives. Finally, we have performed sensitivity analysis and comparative analysis to check the validity of the results in different circumstances.

This model will be very helpful for policymakers to improve the educational facilities and funding allocation. It will help administrators to identify the weaknesses and strengths of an educational institution for enhancing the quality and for taking the necessary steps. There are also some future research scopes. In this paper, we have applied an entropy-VIKOR-based approach, whereas one can use some other effective MCDM methods for the formation of this model. One can further use some other advanced fuzzy number rather than using GPIFNs for numerical evaluation. The number of criteria and alternatives can also be increased to increase the complexity of the problem.

Acknowledgement
This research was not funded by any grant.

Conflicts of Interest

The authors declare no conflicts of interest.

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