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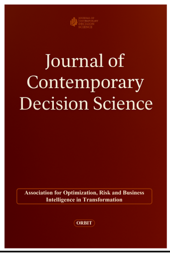

IndetermSoft Sets: A Multi-Valued Framework for Modeling Indeterminacy in Decision Science

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ABSTRACT

Soft sets are parameterized families of subsets of a universe, where each parameter selects elements relevant under a condition. This paper develops *IndetermSoft sets*, a class designed to treat indeterminacy explicitly through structured value slices. We define five variants: (i) Single-valued, mapping each attribute value to one subset capturing undirected indeterminacy; (ii) Double-valued, splitting into truth-leaning and falsity-leaning slices; (iii) Triple-valued, adding a neutral slice; (iv) Quadruple-valued, refining polarity by distinguishing strong and ordinary truth/falsity leanings; and (v) Quintuple-valued, further introducing a central neutral band between the two strengths. These constructions yield a uniform framework for modeling ambiguous evidence, conflicts, and missing context.

1. Preliminaries

We collect the basic terminology and notation used in what follows. The definitions in this paper are assumed to be finite.

1.1 *Soft Set and IndetermSoft Set*

The definitions of the Soft Set and the IndetermSoft Set are provided below. Soft Set is a parameterized family of subsets where each parameter selects relevant elements from a universe for decision modeling [1, 2].

Definition 1.1 (Soft Set)**.** [1] Let U be a universal set and E a set of parameters. A *soft set* over U is

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defined as an ordered pair (F, E), where F is a mapping from E to the power set $\mathcal{P}(U)$:

$$F : E \to \mathcal{P}(U).$$

For each parameter $e \in E$, $F(e) \subseteq U$ represents the set of e-approximate elements in U, with (F, E) forming a parameterized family of subsets of U.

Example 1.2 (Soft Set - Apartment Search by Parameters)**.** A renter filters apartments using simple binary conditions. Let the universe be the set of available listings

$$U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}.$$

Let the parameter set be

$$E = \{\text{near\_subway},\ \text{pet\_friendly},\ \text{balcony},\ \text{under\$1500},\ \text{new\_build}\}.$$

Define a soft set (F, E) with $F : E \to \mathcal{P}(U)$:

$$\begin{aligned} F(\text{near\_subway}) &= \{u_1, u_2, u_5, u_6, u_8\}, \\ F(\text{pet\_friendly}) &= \{u_2, u_3, u_6, u_7\}, \\ F(\text{balcony}) &= \{u_1, u_3, u_4, u_6\}, \\ F(\text{under\$1500}) &= \{u_2, u_4, u_5, u_7\}, \\ F(\text{new\_build}) &= \{u_1, u_5, u_8\}. \end{aligned}$$

Each $F(e) \subseteq U$ collects listings relevant under parameter e.
Concrete queries with explicit computations.

$$\begin{aligned} \text{(i) near\_subway \& under\$1500:} \quad & F(\text{near\_subway}) \cap F(\text{under\$1500}) \\ & = \{u_1, u_2, u_5, u_6, u_8\} \cap \{u_2, u_4, u_5, u_7\} \\ & = \{u_2, u_5\}, \quad |\cdot| = 2. \\ \text{(ii) pet\_friendly or balcony:} \quad & F(\text{pet\_friendly}) \cup F(\text{balcony}) = \{u_2, u_3, u_6, u_7\} \cup \{u_1, u_3, u_4, u_6\} \\ & = \{u_1, u_2, u_3, u_4, u_6, u_7\}, \quad |\cdot| = 6. \\ \text{(iii) new\_build \& balcony:} \quad & F(\text{new\_build}) \cap F(\text{balcony}) = \{u_1, u_5, u_8\} \cap \{u_1, u_3, u_4, u_6\} \\ & = \{u_1\}, \quad |\cdot| = 1. \end{aligned}$$

These set-theoretic combinations show how a soft set supports multi-constraint filtering with exact results.

Single-valued IndetermSoft Set maps each attribute value to one subset capturing undirected, non-unique indeterminacy over H; domain/codomain may be indeterminate [3, 4].

Definition 1.3 ((Single-valued) IndetermSoft set)**.** [5] Let U be a universe of discourse, $H \subseteq U$ a non-empty subset, and $P(H)$ the powerset of H. Let A be the set of attribute values for an attribute a. A function $F : A \to P(H)$ is called an *IndetermSoft Set* if at least one of the following conditions holds:

1. A has some indeterminacy.
2. $P(H)$ has some indeterminacy.
3. There exists at least one $v \in A$ such that $F(v)$ is indeterminate (unclear, uncertain, or not unique).

4. Any two or all three of the above conditions.

An IndetermSoft Set is represented mathematically as:

$$F : A \to H(\cap, \cup, \oplus, \neg),$$

where $H(\cap, \cup, \oplus, \neg)$ represents a structure closed under the IndetermSoft operators.

Example 1.4 ((Single-valued) IndetermSoft Set - Identity Verification Queue). **Context.** A hiring team triages applicants whose identity documents have quality issues. Let the review subset be

$$H = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\} \subseteq U,$$

and let the attribute be "document condition" with values

$$A = \{\text{low\_light\_photo},\ \text{partial\_scan},\ \text{metadata\_mismatch}\}.$$

We define a single-valued IndetermSoft mapping $F : A \to \mathcal{P}(H)$, where $F(v)$ collects cases with *undirected* indeterminacy under v (neither truth- nor falsity-leaning).
Determinately specified slices.

$$F(\text{low\_light\_photo}) = \{a_1, a_3, a_6\}, \quad |F| = 3,$$
$$F(\text{partial\_scan}) = \{a_2, a_4\}, \quad |F| = 2.$$

Indeterminate slice (explicit non-uniqueness). For $v^\star = \text{metadata\_mismatch}$, conflicting HR and ATS logs produce two admissible subsets:

$$S_1 = \{a_5\} \quad \text{(only the strict mismatch)}, \qquad S_2 = \{a_5, a_7\} \quad \text{(strict + borderline timestamp drift)}.$$

Hence $F(v^\star)$ is *not uniquely determined*:

$$F(\text{metadata\_mismatch}) \in \{S_1, S_2\} = \{\{a_5\},\ \{a_5, a_7\}\}.$$

This satisfies condition (3) of the IndetermSoft definition (indeterminate image).
Let the determinate union be

$$L := F(\text{low\_light\_photo}) \cup F(\text{partial\_scan}) = \{a_1, a_2, a_3, a_4, a_6\}.$$

Including the indeterminate value yields an upper bound

$$U := L \cup F(\text{metadata\_mismatch}) \in \{L \cup S_1,\ L \cup S_2\}$$

$$= \Big\{\{a_1, a_2, a_3, a_4, a_5, a_6\},\ \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}\Big\}.$$

Therefore the review queue size is bounded as

$$|L| = 5 \ \le\ |U| \ \in\ \{6, 7\}.$$

This concrete non-uniqueness of $F(v^\star)$ exemplifies a (single-valued) IndetermSoft set: one subset per value, with explicit indeterminacy captured by alternative admissible images.

1.2 Double-Valued Neutrosophic Set

Neutrosophic Set assigns to each element independent degrees of truth, indeterminacy, and falsity, enabling nuanced uncertainty modeling beyond fuzzy sets[6, 7]. Double-Valued Neutrosophic Set records truth, truth-leaning indeterminacy, falsity-leaning indeterminacy, and falsity degrees for each element, capturing polarized ambiguous evidence effectively[8, 9]. We consider Double-Valued Neutrosophic Logic, as outlined below (cf. [8, 10]).

Definition 1.5 (Double-Valued Neutrosophic Set)**.** [8] Let X be a space of points (or objects) where each $x \in X$ represents an element. A *Double-Valued Neutrosophic Set (DVNS)* A is characterized by:

$$A = \{(x, T_A(x), I_T(x), I_F(x), F_A(x)) : x \in X\},$$

where:

- $T_A(x) \in [0, 1]$ is the *truth membership value*,
- $I_T(x) \in [0, 1]$ is the *indeterminacy leaning towards truth*,
- $I_F(x) \in [0, 1]$ is the *indeterminacy leaning towards falsity*,
- $F_A(x) \in [0, 1]$ is the *falsity membership value*.

These values satisfy the condition:

$$0 \leq T_A(x) + I_T(x) + I_F(x) + F_A(x) \leq 4.$$

Example 1.6 (Double-Valued Neutrosophic Set - Phishing Email Screening)**. Universe and target.** Let X be the set of incoming emails and let $A \subseteq [0, 1]^4$ encode a DVNS for the concept *LegitimateMail*. For each email $x \in X$ we assign

$$\big(T_A(x),\ I_T(x),\ I_F(x),\ F_A(x)\big),$$

where T_A is truth (legitimate), F_A is falsity (phishing), I_T leans toward legitimacy, and I_F leans toward phishing.
Signals. We aggregate authentication (SPF/DKIM/DMARC), sender reputation, URL/attachment analysis, and writing-style features. Ambiguous signals (e.g., one pass and one fail) contribute to I_T or I_F; consistent evidence contributes to T_A or F_A.
Concrete assignments (three emails).

email	T_A	I_T	I_F	F_A
e_1	0.72	0.14	0.06	0.08
e_2	0.38	0.20	0.22	0.20
e_3	0.10	0.08	0.27	0.55

Justification. e_1: known domain, DMARC pass $\Rightarrow$ large T_A; single shortened URL $\Rightarrow$ small I_F. e_2: mixed signals (fresh domain, but no payload) $\Rightarrow$ comparable I_T and I_F. e_3: macro-enabled attachment and domain spoof patterns $\Rightarrow$ large F_A.
Numerical verification. Row sums:

$$e_1 : 0.72 + 0.14 + 0.06 + 0.08 = 1.00 \leq 4,$$
$$e_2 : 0.38 + 0.20 + 0.22 + 0.20 = 1.00 \leq 4,$$
$$e_3 : 0.10 + 0.08 + 0.27 + 0.55 = 1.00 \leq 4.$$

Thus each quadruple satisfies the DVNS constraint $0 \leq T_A + I_T + I_F + F_A \leq 4$.

1.3 Triple-Valued Neutrosophic Set (TVNS)

A Triple-Valued Neutrosophic Set assigns each element degrees for truth, truth-leaning indeterminacy, neutral indeterminacy, falsity-leaning indeterminacy, and falsity, within limits. In this subsection, we examine the concept of the Triple-Valued Neutrosophic Set (TVNS) [11, 12].

Definition 1.7 (Triple-Valued Neutrosophic Set (TVNS))**.** [12] A *Triple-Valued Neutrosophic Set* A on X is defined as

$$A = \Big\{ \Big(x, T_A(x), I_T(x), I_N(x), I_F(x), F_A(x) \Big) : x \in X \Big\},$$

where

- $T_A(x) \in [0, 1]$ is the truth membership degree,
- $I_T(x) \in [0, 1]$ is the *indeterminacy leaning towards truth*,
- $I_N(x) \in [0, 1]$ is the *neutral indeterminacy* (i.e., completely indeterminate, neither leaning towards truth nor falsity),
- $I_F(x) \in [0, 1]$ is the *indeterminacy leaning towards falsity*,
- $F_A(x) \in [0, 1]$ is the falsity membership degree.

For each $x \in X$, we have

$$0 \le T_A(x) + I_T(x) + I_N(x) + I_F(x) + F_A(x) \le 5.$$

Example 1.8 (Triple-Valued Neutrosophic Set - Loan Approval Under Missing Evidence)**. Universe and target.** Let X be the set of loan applications and consider the TVNS for the concept *Creditworthy*. For each applicant $x \in X$ we assign

$$\big(T_A(x),\ I_T(x),\ I_N(x),\ I_F(x),\ F_A(x)\big),$$

where I_N is *neutral* indeterminacy due to missing or non-informative evidence.
Evidence channels. Credit score and repayment history (push T_A), soft positives such as stable address (push I_T), missing employer verification or frozen bureau file (push I_N), soft negatives like recent inquiries (push I_F), and hard negatives such as charge-offs (push F_A).
Concrete assignments (three applicants).

applicant	T_A	I_T	I_N	I_F	F_A
a_1	0.55	0.15	0.10	0.08	0.12
a_2	0.32	0.18	0.30	0.10	0.10
a_3	0.12	0.08	0.20	0.25	0.35

Justification. a_1: strong score and verified income $\Rightarrow$ larger T_A; a minor document issue $\Rightarrow$ small I_N. a_2: acceptable score but employer response pending $\Rightarrow$ elevated I_N; mixed soft signals split between I_T and I_F. a_3: prior delinquency and high utilization $\Rightarrow$ large F_A; some uncollected data adds to I_N.
Numerical verification. Row sums:

$$a_1 : 0.55 + 0.15 + 0.10 + 0.08 + 0.12 = 1.00 \le 5,$$
$$a_2 : 0.32 + 0.18 + 0.30 + 0.10 + 0.10 = 1.00 \le 5,$$
$$a_3 : 0.12 + 0.08 + 0.20 + 0.25 + 0.35 = 1.00 \le 5.$$

Hence each quintuple satisfies the TVNS constraint $0 \le T_A + I_T + I_N + I_F + F_A \le 5$.

1.4 Quadruple-Valued Neutrosophic Set (QVNS)

A Quadruple-Valued Neutrosophic Set assigns each element truth, strongly truth-leaning indeterminacy, truth-leaning indeterminacy, falsity-leaning indeterminacy, strongly falsity-leaning indeterminacy, and falsity. The definition of the Quadruple-Valued Neutrosophic Set (QVNS) is provided below [12, 13].

Definition 1.9 (Quadruple-Valued Neutrosophic Set (QVNS)). [12] Let X be a universe of discourse. A *Quadruple-Valued Neutrosophic Set* A on X is defined as

$$A = \Big\{ \big(x,\, T_A(x),\, I_T^s(x),\, I_T(x),\, I_F(x),\, I_F^s(x),\, F_A(x)\big) : x \in X \Big\},$$

where:

- $T_A(x) \in [0,1]$ is the *truth membership degree*,
- $I_T^s(x) \in [0,1]$ is the *indeterminacy strongly leaning towards truth*,
- $I_T(x) \in [0,1]$ is the *indeterminacy leaning towards truth*,
- $I_F(x) \in [0,1]$ is the *indeterminacy leaning towards falsity*,
- $I_F^s(x) \in [0,1]$ is the *indeterminacy strongly leaning towards falsity*,
- $F_A(x) \in [0,1]$ is the *falsity membership degree*.

For each $x \in X$, it holds that

$$0 \le T_A(x) + I_T^s(x) + I_T(x) + I_F(x) + I_F^s(x) + F_A(x) \le 6.$$

Example 1.10 (Quadruple-Valued Neutrosophic Set - E-commerce Product Authenticity). **Universe and target.** Let X be items listed on a marketplace. Consider the QVNS for the concept *Authentic-Item*. For each item $x \in X$ we assign

$$\big(T_A(x),\, I_T^s(x),\, I_T(x),\, I_F(x),\, I_F^s(x),\, F_A(x)\big),$$

where T_A/F_A are truth/falsity memberships (authentic vs. counterfeit), and I_T^s, I_T, I_F, I_F^s are indeterminacy slices (strong/ordinary truth- or falsity-leaning).

Evidence channels. Manufacturer serial check (pushes I_T^s or T_A when strongly positive), invoice validation (pushes I_T), packaging/label anomalies (push I_F), known counterfeit patterns (push I_F^s), and third-party lab confirmation (boosts T_A or F_A).

Concrete assignments (three items).

item	T_A	I_T^s	I_T	I_F	I_F^s	F_A
i_1	0.60	0.15	0.10	0.05	0.03	0.07
i_2	0.30	0.05	0.20	0.25	0.10	0.10
i_3	0.08	0.02	0.10	0.25	0.35	0.20

Justification. i_1: serial verified and lab spot-check $\Rightarrow$ large T_A, some I_T^s; a minor label scuff $\Rightarrow$ small I_F. i_2: mixed invoice and seller history $\Rightarrow$ comparable I_T and I_F; moderate T_A. i_3: hallmark counterfeit indicators $\Rightarrow$ large I_F^s and F_A.

Numerical verification. For each row,

$$i_1 : 0.60 + 0.15 + 0.10 + 0.05 + 0.03 + 0.07 = 1.00 \le 6,$$
$$i_2 : 0.30 + 0.05 + 0.20 + 0.25 + 0.10 + 0.10 = 1.00 \le 6,$$
$$i_3 : 0.08 + 0.02 + 0.10 + 0.25 + 0.35 + 0.20 = 1.00 \le 6.$$

Thus each sextuple satisfies $0 \le T_A + I_T^s + I_T + I_F + I_F^s + F_A \le 6$.

1.5 Quintuple-Valued Neutrosophic Set (QnVNS)

A Quintuple-Valued Neutrosophic Set assigns each element truth; strong truth-leaning, truth-leaning, neutral, falsity-leaning, strong falsity-leaning indeterminacies; and falsity, with bounds. The definition of the Quintuple-Valued Neutrosophic Set (QnVNS) is provided below. [12, 13]

Definition 1.11 (Quintuple-Valued Neutrosophic Set (QnVNS))**.** [12] Let X be a universe of discourse. A *Quintuple-Valued Neutrosophic Set* A on X is defined as

$$A = \Big\{ \big(x,\, T_A(x),\, I_T^s(x),\, I_T(x),\, I_N(x),\, I_F(x),\, I_F^s(x),\, F_A(x)\big) : x \in X \Big\},$$

where:

- $T_A(x) \in [0,1]$ is the *truth membership degree*,
- $I_T^s(x) \in [0,1]$ is the *indeterminacy strongly leaning towards truth*,
- $I_T(x) \in [0,1]$ is the *indeterminacy leaning towards truth*,
- $I_N(x) \in [0,1]$ represents *neutral indeterminacy* (i.e., completely indeterminate, neither leaning towards truth nor falsity),
- $I_F(x) \in [0,1]$ is the *indeterminacy leaning towards falsity*,
- $I_F^s(x) \in [0,1]$ is the *indeterminacy strongly leaning towards falsity*,
- $F_A(x) \in [0,1]$ is the *falsity membership degree*.

For each $x \in X$, it holds that

$$0 \le T_A(x) + I_T^s(x) + I_T(x) + I_N(x) + I_F(x) + I_F^s(x) + F_A(x) \le 7.$$

Example 1.12 (Quintuple-Valued Neutrosophic Set - Product Recall Impact Assessment)**. Universe and target.** Let X be purchased product units. Consider the QnVNS for the concept *AffectedByRecall*. For each unit $x \in X$ we assign

$$\big(T_A(x),\, I_T^s(x),\, I_T(x),\, I_N(x),\, I_F(x),\, I_F^s(x),\, F_A(x)\big),$$

where I_N is *neutral* indeterminacy (e.g., missing batch code).
Evidence channels. Lot/batch match (push I_T^s/T_A), weak cues such as packaging date proximity (push I_T), missing or unreadable label (push I_N), retailer confirmation of safe lot (push I_F), and manufacturer certificate of unaffected series (push I_F^s/F_A).
Concrete assignments (three units).

unit	T_A	I_T^s	I_T	I_N	I_F	I_F^s	F_A
b_1	0.55	0.20	0.10	0.05	0.05	0.02	0.03
b_2	0.20	0.05	0.15	0.30	0.15	0.05	0.10
b_3	0.08	0.02	0.10	0.10	0.20	0.25	0.25

Justification. b_1: exact lot match and vendor alert ⇒ large T_A and I_T^s. b_2: label unreadable ⇒ elevated I_N; weak context on both sides. b_3: certificate proves different series ⇒ higher I_F^s and F_A.
Numerical verification. For each row,

$$b_1 : 0.55 + 0.20 + 0.10 + 0.05 + 0.05 + 0.02 + 0.03 = 1.00 \le 7,$$
$$b_2 : 0.20 + 0.05 + 0.15 + 0.30 + 0.15 + 0.05 + 0.10 = 1.00 \le 7,$$
$$b_3 : 0.08 + 0.02 + 0.10 + 0.10 + 0.20 + 0.25 + 0.25 = 1.00 \le 7.$$

Hence each septuple satisfies $0 \le T_A + I_T^s + I_T + I_N + I_F + I_F^s + F_A \le 7$.

1.6 Double-valued IndetermSoft Set

Double-valued IndetermSoft Set maps each attribute value to two subsets: truth-leaning indeterminacy and falsity-leaning indeterminacy within a universe, allowing overlaps. The definition of the Double-valued IndetermSoft Set is provided below. [12]

Definition 1.13 (Double-valued IndetermSoft Set)**.** [12] Let U be a universe of discourse, $H \subseteq U$ a non-empty subset, and $\mathcal{P}(H)$ the power set of H. Let A be a set of attribute values associated with a specific attribute a. A function

$$F : A \to \mathcal{P}(H) \times \mathcal{P}(H)$$

is called a *Double-valued IndetermSoft Set* if any of the following conditions hold:

1. The set A exhibits some degree of indeterminacy.
2. The power set $\mathcal{P}(H)$ exhibits some level of indeterminacy.
3. There exists at least one attribute value $v \in A$ such that the ordered pair

$$F(v) = \big(F_T(v),\, F_F(v)\big)$$

   is indeterminate, in the sense that either $F_T(v)$ or $F_F(v)$ (or both) is unclear, uncertain, or not uniquely defined.
4. Any combination of the above conditions holds.

Here, for each $v \in A$,

- $F_T(v) \subseteq H$ represents the *indeterminacy leaning towards truth* associated with v, and
- $F_F(v) \subseteq H$ represents the *indeterminacy leaning towards falsity* associated with v.

Example 1.14 (Double-valued IndetermSoft Set - Warranty Claims Triage)**.** Let the universe H be a finite set of consumer-electronics warranty claims

$$H = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}.$$

Fix an attribute “document reliability,” with attribute values

$$A = \{\text{blurred\_receipt},\ \text{third\_party\_repair},\ \text{serial\_OCR\_error}\}.$$

Define a Double-valued IndetermSoft Set $F : A \to \mathcal{P}(H) \times \mathcal{P}(H)$ by

$$\begin{aligned} F(\text{blurred\_receipt}) &= \big(F_T, F_F\big) = \big(\{c_1, c_2, c_5\},\ \{c_3\}\big), \\ F(\text{third\_party\_repair}) &= \big(\{c_5\},\ \{c_4, c_6\}\big), \\ F(\text{serial\_OCR\_error}) &= \big(\{c_2, c_6\},\ \{c_7\}\big). \end{aligned}$$

Interpretation.

- $F_T(v)$ (truth-leaning indeterminacy): claims whose evidence *likely* supports validity under v, but is still indeterminate (e.g., other documents corroborate a blurred receipt).
- $F_F(v)$ (falsity-leaning indeterminacy): claims whose evidence *likely* undermines validity under v (e.g., non-authorized third-party repair), yet not conclusively.

Concrete checks (cardinalities and overlaps across different v):

$$|F_T(\text{blurred\_receipt})| = 3, \quad |F_F(\text{blurred\_receipt})| = 1, \quad |F_T \cup F_F| = 4,$$
$$|F_T(\text{third\_party\_repair})| = 1, \quad |F_F(\text{third\_party\_repair})| = 2, \quad |F_T \cup F_F| = 3,$$
$$|F_T(\text{serial\_OCR\_error})| = 2, \quad |F_F(\text{serial\_OCR\_error})| = 1, \quad |F_T \cup F_F| = 3.$$

Cross-attribute tension is visible: for instance, $c_6 \in F_F(\text{third\_party\_repair})$ but $c_6 \in F_T(\text{serial\_OCR\_error})$, capturing conflicting indeterminate signals that motivate a second-stage manual review.

2. Main Results

In this section, we present the main contributions of this paper.

2.1 Triple-valued IndetermSoft Set

Triple-valued IndetermSoft Set maps each attribute value to three subsets: truth-leaning, neutral, and falsity-leaning indeterminacy over a universe, allowing overlaps.

Definition 2.1 (Triple-valued IndetermSoft Set (TV-ISS))**.** Let U be a universe, $H \subseteq U$ nonempty, and A a set of attribute values. A TV-ISS is a mapping

$$G : A \longrightarrow \mathcal{P}(H)^3, \qquad G(v) = \big(G_T(v),\, G_N(v),\, G_F(v)\big),$$

with the following intended semantics for each $v \in A$:

- $G_T(v) \subseteq H$: indeterminacy leaning to truth,
- $G_N(v) \subseteq H$: *neutral* indeterminacy (neither leaning to truth nor to falsity),
- $G_F(v) \subseteq H$: indeterminacy leaning to falsity.

No exclusivity is imposed a priori; intersections are allowed if the application dictates. When needed, one may assume $G_T(v), G_N(v), G_F(v)$ are pairwise disjoint to model a partition of the indeterminate region.

Example 2.2 (Concrete TV-ISS, its projection, and the embedding)**.** Let the finite universe and attribute-values be

$$H = \{h_1, h_2, h_3, h_4\}, \qquad A = \{\text{easy}, \text{hard}\}.$$

Define a TV-ISS $G : A \to \mathcal{P}(H)^3$ by

$$G(\text{easy}) = \big(\ \{h_1, h_2\},\ \{h_3\},\ \{h_4\}\ \big),$$
$$G(\text{hard}) = \big(\ \{h_1\},\ \{h_2, h_3\},\ \{h_4\}\ \big).$$

Cardinalities are

$$|G_T(\text{easy})| = 2, \quad |G_N(\text{easy})| = 1, \quad |G_F(\text{easy})| = 1,$$
$$|G_T(\text{hard})| = 1, \quad |G_N(\text{hard})| = 2, \quad |G_F(\text{hard})| = 1.$$

The canonical DV-ISS projection $\Pi(G)$ and upper envelope $\overline{\Pi}(G)$ are

$$\Pi(G)(\text{easy}) = \big(\{h_1, h_2\},\ \{h_4\}\big), \quad \overline{\Pi}(G)(\text{easy}) = \big(\{h_1, h_2, h_3\},\ \{h_3, h_4\}\big),$$
$$\Pi(G)(\text{hard}) = \big(\{h_1\},\ \{h_4\}\big), \qquad \overline{\Pi}(G)(\text{hard}) = \big(\{h_1, h_2, h_3\},\ \{h_2, h_3, h_4\}\big),$$

and one checks the coordinatewise inclusions

$$\Pi(G) \preceq \overline{\Pi}(G),$$

since, for instance, $\{h_1, h_2\} \subseteq \{h_1, h_2, h_3\}$ and $\{h_4\} \subseteq \{h_3, h_4\}$ at $v = \text{easy}$.
Now apply the DV→TV embedding ι to the DV-ISS $F := \Pi(G)$:

$$\big(\iota(F)\big)(\text{easy}) = \big(\{h_1, h_2\},\ \varnothing,\ \{h_4\}\big), \qquad \big(\iota(F)\big)(\text{hard}) = \big(\{h_1\},\ \varnothing,\ \{h_4\}\big).$$

By construction,

$$(\Pi \circ \iota)(F) = F$$

holds *pointwise* and *componentwise*, e.g.,

$$(\Pi \circ \iota)(F)(\text{easy}) = \Pi\big(\{h_1, h_2\}, \varnothing, \{h_4\}\big) = \big(\{h_1, h_2\}, \{h_4\}\big) = F(\text{easy}).$$

This verifies, with explicit sets and equalities, the identity $\Pi \circ \iota = \text{id}$ from Theorem 2.4.

Example 2.3 (Triple-valued IndetermSoft Set - Hiring Degree Verification)**.** Let the universe H be a finite set of applicants

$$H = \{a_1, a_2, a_3, a_4, a_5, a_6\}.$$

Consider the attribute "credential verification," with attribute values

$$A = \{\text{degree\_email\_unreplied},\ \text{transcript\_low\_quality}\}.$$

Define a Triple-valued IndetermSoft Set $G : A \to \mathcal{P}(H)^3$ by

$$G(\text{degree\_email\_unreplied}) = \big(G_T, G_N, G_F\big) = \big(\{a_1, a_3\},\ \{a_2, a_5\},\ \{a_6\}\big),$$
$$G(\text{transcript\_low\_quality}) = \big(\{a_3, a_4\},\ \{a_5\},\ \{a_2\}\big).$$

Semantics for each $v \in A$:

- $G_T(v)$: truth-leaning indeterminacy (signals suggest the claim is likely true but not definitive yet; e.g., alumni registry hints).
- $G_N(v)$: neutral indeterminacy (insufficient information; e.g., awaiting registrar reply).
- $G_F(v)$: falsity-leaning indeterminacy (signals suggest likely false; e.g., domain mismatch, inconsistent graduation dates).

Concrete verifications (sizes and disjointness here by design):

$$|G_T(\text{degree\_email\_unreplied})| = 2, \quad |G_N| = 2, \quad |G_F| = 1, \quad |G_T \cup G_N \cup G_F| = 5,$$
$$|G_T(\text{transcript\_low\_quality})| = 2, \quad |G_N| = 1, \quad |G_F| = 1, \quad |G_T \cup G_N \cup G_F| = 4.$$

If one wishes to compare with a Double-valued view, the canonical projection that discards the neutral slice is

$$F^{\downarrow}(v) := \big(G_T(v),\ G_F(v)\big),$$

so, for example, $F^{\downarrow}(\text{degree\_email\_unreplied}) = (\{a_1, a_3\}, \{a_6\})$, which is ready for two-queue downstream handling (provisional accept vs. provisional reject) while cases in G_N await additional evidence.

Theorem 2.4 (TV-ISS generalizes DV-ISS). *There exist mappings*

$$\iota : \mathrm{ISS}_2(A,H) \to \mathrm{ISS}_3(A,H), \qquad \Pi : \mathrm{ISS}_3(A,H) \to \mathrm{ISS}_2(A,H)$$

defined pointwise by

$$\big(\iota(F)\big)(v) \;:=\; \big(F_T(v),\, \varnothing,\, F_F(v)\big), \qquad \big(\Pi(G)\big)(v) \;:=\; \big(G_T(v),\, G_F(v)\big),$$

such that

$$\Pi \circ \iota \;=\; \mathrm{id}_{\mathrm{ISS}_2(A,H)} \,.$$

Consequently, $\mathrm{ISS}_2(A,H)$ *is isomorphic to the sub-class*

$$\mathrm{ISS}_3^{(0)}(A,H) \;:=\; \Big\{ G \in \mathrm{ISS}_3(A,H) \;:\; G_N(v) = \varnothing \text{ for all } v \in A \Big\} \subseteq \mathrm{ISS}_3(A,H),$$

and every DV-ISS is a special case of a TV-ISS (obtained by taking all neutral slices empty).

Proof. Fix $F \in \mathrm{ISS}_2(A,H)$. By definition,

$$F : A \to \mathcal{P}(H) \times \mathcal{P}(H), \qquad v \mapsto \big(F_T(v), F_F(v)\big).$$

Define $\iota(F) : A \to \mathcal{P}(H)^3$ by $\big(\iota(F)\big)(v) = \big(F_T(v), \varnothing, F_F(v)\big)$. This lies in $\mathrm{ISS}_3(A,H)$ since $\varnothing \in \mathcal{P}(H)$.
Next, for $G \in \mathrm{ISS}_3(A,H)$ with $G(v) = (G_T(v), G_N(v), G_F(v))$, define $\Pi(G)(v) = (G_T(v), G_F(v))$. Then $\Pi(G) \in \mathrm{ISS}_2(A,H)$.
We compute the composition explicitly, for each $v \in A$:

$$\big((\Pi \circ \iota)(F)\big)(v) \;=\; \Pi\big(\iota(F)\big)(v) = \; \Pi\big(F_T(v), \varnothing, F_F(v)\big) = \; \big(F_T(v), F_F(v)\big) = \; F(v).$$

Hence $\Pi \circ \iota = \mathrm{id}_{\mathrm{ISS}_2(A,H)}$.
Injectivity of ι: if $\iota(F) = \iota(F')$, then for every v,

$$\big(F_T(v), \varnothing, F_F(v)\big) = \big(F'_T(v), \varnothing, F'_F(v)\big),$$

whence $F_T(v) = F'_T(v)$ and $F_F(v) = F'_F(v)$, so $F = F'$.
Moreover, the image of ι is precisely $\mathrm{ISS}_3^{(0)}(A,H)$. Indeed, for any $G \in \mathrm{ISS}_3^{(0)}(A,H)$ one has $G_N(v) = \varnothing$ for all v, and setting $F(v) = (G_T(v), G_F(v))$ yields $G = \iota(F)$.
Therefore DV-ISS is canonically identified with the neutral-empty sub-class of TV-ISS, so TV-ISS strictly generalizes DV-ISS. □

2.2 *Quadruple-valued IndetermSoft Set*

Quadruple-valued IndetermSoft Set assigns *four* indeterminacy slices per attribute value, mirroring Quadruple-valued Neutrosophic:

$$\text{(strong truth-leaning)},\ \text{truth-leaning},\ \text{falsity-leaning},\ \text{strong falsity-leaning}.$$

Definition 2.5 (Quadruple-valued IndetermSoft Set (QV-ISS)). Let U be a universe, $H \subseteq U$ nonempty, and A a set of attribute values. A QV-ISS is a mapping

$$Q : A \longrightarrow \mathcal{P}(H)^4, \qquad Q(v) = \big(Q_T^s(v),\ Q_T(v),\ Q_F(v),\ Q_F^s(v)\big),$$

where, for each $v \in A$:

- $Q^s_T(v) \subseteq H$: **strong truth-leaning indeterminacy** (very strong evidence toward truth, yet not fully resolved),
- $Q_T(v) \subseteq H$: **truth-leaning indeterminacy** (moderate evidence toward truth, still inconclusive),
- $Q_F(v) \subseteq H$: **falsity-leaning indeterminacy** (moderate evidence toward falsity, still inconclusive),
- $Q^s_F(v) \subseteq H$: **strong falsity-leaning indeterminacy** (very strong evidence toward falsity, yet not fully resolved).

Theorem 2.6 (QV-ISS generalizes TV-ISS)**.** *Define maps*

$$\iota:\ (\mathcal{P}(H)^3)^A \longrightarrow (\mathcal{P}(H)^4)^A, \qquad \big(\iota(G)\big)(v) := \big(\varnothing,\ G_T(v)\cup G_N(v),\ G_F(v)\cup G_N(v),\ \varnothing\big),$$

and

$$\Pi:\ (\mathcal{P}(H)^4)^A \longrightarrow (\mathcal{P}(H)^3)^A, \qquad \big(\Pi(Q)\big)(v) := \big(\,T(v),\ N(v),\ F(v)\,\big),$$

where, writing $S_T(v) := Q^s_T(v) \cup Q_T(v)$ *and* $S_F(v) := Q^s_F(v) \cup Q_F(v)$,

$$T(v) := S_T(v) \setminus S_F(v), \qquad N(v) := S_T(v) \cap S_F(v), \qquad F(v) := S_F(v) \setminus S_T(v).$$

Then for every TV-ISS G in partition form, one has

$$(\Pi \circ \iota)(G) \;=\; G.$$

Consequently, TV-ISS embeds injectively into QV-ISS; hence QV-ISS (canonically) generalizes TV-ISS.

Proof. Fix G and $v \in A$. By definition of ι,

$$Q^s_T = \varnothing, \quad Q_T = G_T \cup G_N, \quad Q_F = G_F \cup G_N, \quad Q^s_F = \varnothing.$$

Hence

$$S_T = Q^s_T \cup Q_T = G_T \cup G_N, \qquad S_F = Q^s_F \cup Q_F = G_F \cup G_N.$$

Using the pairwise disjointness $G_T \cap G_N = G_N \cap G_F = G_T \cap G_F = \varnothing$, we compute:

$$T = S_T \setminus S_F = (G_T \cup G_N) \setminus (G_F \cup G_N) = \big(G_T \setminus (G_F \cup G_N)\big) \;\cup\; \underbrace{(G_N \setminus (G_F \cup G_N))}_{=\varnothing} = G_T,$$

$$N = S_T \cap S_F = (G_T \cup G_N) \cap (G_F \cup G_N) = \underbrace{(G_T \cap G_F)}_{=\varnothing} \cup \underbrace{(G_T \cap G_N)}_{=\varnothing} \cup \underbrace{(G_N \cap G_F)}_{=\varnothing} \cup G_N = G_N,$$

$$F = S_F \setminus S_T = (G_F \cup G_N) \setminus (G_T \cup G_N) = \big(G_F \setminus (G_T \cup G_N)\big) \;\cup\; \underbrace{(G_N \setminus (G_T \cup G_N))}_{=\varnothing} = G_F.$$

Therefore $(\Pi \circ \iota)(G)(v) = (G_T(v), G_N(v), G_F(v)) = G(v)$ for every $v \in A$, i.e., $(\Pi \circ \iota)(G) = G$. Since $\Pi \circ \iota = \mathrm{id}$ on TV-ISS, ι is injective, providing an embedding of TV-ISS into QV-ISS. □

2.3 Quintuple-valued IndetermSoft Set

Quintuple-valued IndetermSoft Set assigns *five* indeterminacy slices per value, mirroring Quintuple-valued Neutrosophic:

$$\text{(strong truth-leaning)},\ \text{truth-leaning},\ \text{neutral},\ \text{falsity-leaning},\ \text{(strong falsity-leaning)}.$$

Definition 2.7 (Quintuple-valued IndetermSoft Set (5V-ISS))**.** A 5V-ISS is a mapping

$$R : A \to \mathcal{P}(H)^5, \qquad R(v) = \big(R_T^s(v),\ R_T(v),\ R_N(v),\ R_F(v),\ R_F^s(v)\big),$$

with semantics, for each $v \in A$:

- $R_T^s(v) \subseteq H$: **strong truth-leaning indeterminacy**,
- $R_T(v) \subseteq H$: **truth-leaning indeterminacy**,
- $R_N(v) \subseteq H$: **neutral indeterminacy** (neither leaning toward truth nor falsity),
- $R_F(v) \subseteq H$: **falsity-leaning indeterminacy**,
- $R_F^s(v) \subseteq H$: **strong falsity-leaning indeterminacy**.

Theorem 2.8 (5V-ISS generalizes QV-ISS)**.** *Define the maps*

$$\iota : \mathrm{ISS}_4(A,H) \to \mathrm{ISS}_5(A,H), \quad \big(\iota(Q)\big)(v) := \big(Q_T^s(v),\, Q_T(v),\, \varnothing,\, Q_F(v),\, Q_F^s(v)\big),$$

$$\Pi : \mathrm{ISS}_5(A,H) \to \mathrm{ISS}_4(A,H), \quad \big(\Pi(R)\big)(v) := \big(R_T^s(v),\, R_T(v),\, R_F(v),\, R_F^s(v)\big).$$

Then

$$\Pi \circ \iota = \mathrm{id}_{\mathrm{ISS}_4(A,H)}.$$

Consequently, $\mathrm{ISS}_4(A,H)$ *is canonically isomorphic to the subclass*

$$\mathrm{ISS}_5^{(0)}(A,H) := \big\{\, R \in \mathrm{ISS}_5(A,H) : R_N(v) = \varnothing \text{ for all } v \in A \,\big\} \subset \mathrm{ISS}_5(A,H),$$

hence 5V-ISS strictly extends QV-ISS whenever some $R_N(v) \neq \varnothing$*.*

Proof. Let $Q \in \mathrm{ISS}_4(A,H)$ and $v \in A$. By definition of ι and Π,

$$(\Pi \circ \iota)(Q)(v) = \Pi\big(Q_T^s(v),\, Q_T(v),\, \varnothing,\, Q_F(v),\, Q_F^s(v)\big) = \big(Q_T^s(v),\, Q_T(v),\, Q_F(v),\, Q_F^s(v)\big) = Q(v).$$

Thus $\Pi \circ \iota = \mathrm{id}$ pointwise and componentwise. Injectivity of ι: if $\iota(Q) = \iota(Q')$, then for every v

$$(Q_T^s, Q_T, \varnothing, Q_F, Q_F^s) = (Q_T^{s\prime}, Q'_T, \varnothing, Q'_F, Q_F^{s\prime}),$$

which forces $Q = Q'$. The image of ι is exactly $\mathrm{ISS}_5^{(0)}(A,H)$ because $R \in \mathrm{ISS}_5(A,H)$ belongs to $\mathrm{ISS}_5^{(0)}(A,H)$ iff $R_N \equiv \varnothing$, in which case $R = \iota(Q)$ with $Q(v) = (R_T^s(v), R_T(v), R_F(v), R_F^s(v))$. To see the extension is strict, fix any $h^\star \in H$ and $v^\star \in A$ and define $R \in \mathrm{ISS}_5(A,H)$ by $R_N(v^\star) = \{h^\star\}$ and the remaining coordinates as in $\iota(Q)$ for some Q; then $R \notin \mathrm{ISS}_5^{(0)}(A,H)$, so R is not in the image of ι. □

Example 2.9 (Concrete embedding and equality check with counts). Let

$$H = \{h_1, h_2, h_3, h_4, h_5\}, \qquad A = \{\text{easy}, \text{hard}\}.$$

Define a QV-ISS $Q \in \mathrm{ISS}_4(A, H)$ by

$$Q(\text{easy}) = \big(\ \{h_1\},\ \{h_2\},\ \{h_4\},\ \{h_5\}\ \big),$$
$$Q(\text{hard}) = \big(\ \{h_1, h_3\},\ \{h_2\},\ \{h_4\},\ \{h_5\}\ \big).$$

Cardinalities:

$$|Q_T^s(\text{easy})| = 1,\ |Q_T(\text{easy})| = 1,\ |Q_F(\text{easy})| = 1,\ |Q_F^s(\text{easy})| = 1;$$
$$|Q_T^s(\text{hard})| = 2,\ |Q_T(\text{hard})| = 1,\ |Q_F(\text{hard})| = 1,\ |Q_F^s(\text{hard})| = 1.$$

Embed via ι to obtain a 5V-ISS $R := \iota(Q)$:

$$R(\text{easy}) = \big(\{h_1\},\ \{h_2\},\ \varnothing,\ \{h_4\},\ \{h_5\}\big),$$
$$R(\text{hard}) = \big(\{h_1, h_3\},\ \{h_2\},\ \varnothing,\ \{h_4\},\ \{h_5\}\big).$$

Project back:

$$\Pi(R)(\text{easy}) = \big(\{h_1\}, \{h_2\}, \{h_4\}, \{h_5\}\big) = Q(\text{easy}),$$
$$\Pi(R)(\text{hard}) = \big(\{h_1, h_3\}, \{h_2\}, \{h_4\}, \{h_5\}\big) = Q(\text{hard}).$$

Hence $(\Pi \circ \iota)(Q) = Q$ with *literal* set equality in all coordinates. To witness strict extension, define R' by taking $R'_N(\text{easy}) = \{h_3\}$ and all other coordinates equal to R; then $R' \notin \mathrm{ISS}_5^{(0)}(A, H)$, so there is no Q' with $R' = \iota(Q')$.

3. Conclusion

In this paper, we defined five variants: (i) **Single-valued**, mapping each attribute value to one subset capturing undirected indeterminacy; (ii) **Double-valued**, splitting into truth-leaning and falsity-leaning slices; (iii) **Triple-valued**, adding a neutral slice; (iv) **Quadruple-valued**, refining polarity by distinguishing strong and ordinary truth/falsity leanings; and (v) **Quintuple-valued**, further introducing a central neutral band between the two strengths. For clarity, the overall summary of the IndetermSoft sets discussed in this paper is presented in Table 1.

In the future, we intend to extend the concepts developed in this paper by employing HyperSoft Sets[14], SuperHyperSoft Sets[15], TreeSoft Sets[16, 17], ForestSoft Sets[18], HyperGraphs[19, 20], and SuperHyperGraphs[21, 22].

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Table 1
Overview of IndetermSoft Set variants (per attribute value)

Variant	Mapping	Indeterminacy slices	Notes / Generalization
Single-valued	$F : A \to \mathcal{P}(H)$	undirected	Minimal form: one subset capturing ambiguity without truth/falsity polarity.
Double-valued	$F : A \to \mathcal{P}(H)^2$	T-leaning; F-leaning	Splits ambiguity by polarity; allows intersections for conflicting signals.
Triple-valued	$G : A \to \mathcal{P}(H)^3$	T-leaning; neutral; F-leaning	Extends Double by adding a neutral band; projects to Double by dropping neutral.
Quadruple-valued	$Q : A \to \mathcal{P}(H)^4$	strong-T; T; F; strong-F	Refines polarity strength; recovers Triple by merging strong/ordinary on each side.
Quintuple-valued	$R : A \to \mathcal{P}(H)^5$	strong-T; T; neutral; F; strong-F	Strict extension of Quadruple; inserts neutral between strengths for finer control.

Author's Contributions

Conceptualization, Takaaki Fujita; Investigation, Takaaki Fujita; Methodology, Takaaki Fujita; Writing – original draft, Takaaki Fujita; Writing – review & editing, All authors.

Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Use of Generative AI and AI-Assisted Tools

I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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